AD-A150 132 APPLICATION OF THE SYSTEM IDENTIFICATION TECHNIQUE TO GOAL-DIRECTED SACCA.. (U) NORTHEASTERN UNIV BOSTON MASS DEPT OF ELECTRICAL ENGINEERING.. J D ENDERLE 30 JUL 84 1/1

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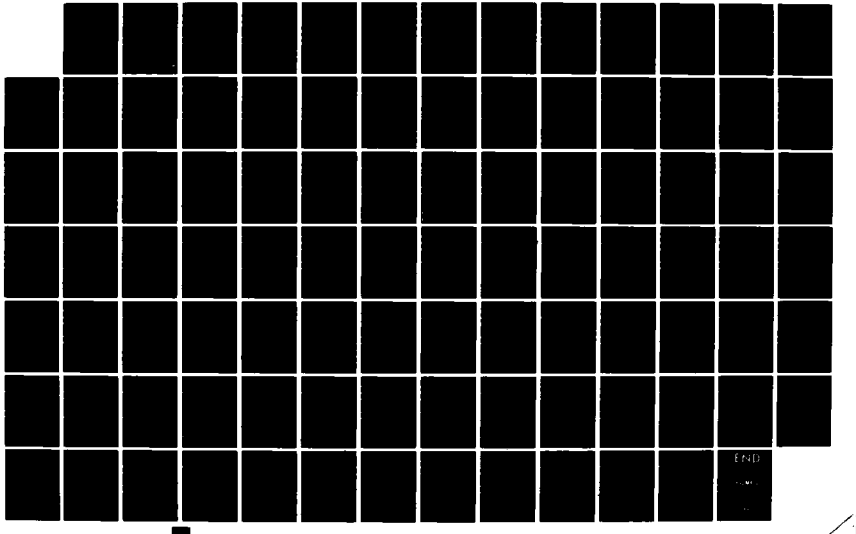

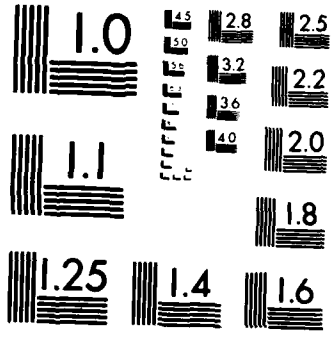

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SECURITY CLASSIFICATION OF THIS PAGE

AD-A150 132

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): 6633 USAF AFOSR-83-0187	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-
6a. NAME OF PERFORMING ORGANIZATION: John D. Enderle, Department of Electrical Engr. / 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: USAF AFOSR
6c. ADDRESS (City, State and ZIP Code): North Dakota State University, Fargo, ND 58105	7b. ADDRESS (City, State and ZIP Code): Department of Air Force, Air Force Office of Scientific Research, Bolling Air Force Base, DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR / 8b. OFFICE SYMBOL (If applicable): NL	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-83-0187
8c. ADDRESS (City, State and ZIP Code): Bolling AFB DC 20332	10. SOURCE OF FUNDING NOS: PROGRAM ELEMENT NO 61102F; PROJECT NO 2313; TASK NO D9; WORK UNIT NO

11. TITLE (Include Security Classification): Application of the System Identification Technique to Goal-Directe Saccades (Unclassified)

12. PERSONAL AUTHOR(S): Enderle, John Denis

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final	FROM 83/6/1 TO 84/5/31	84/7/30	93

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB GR

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number): System Identification Technique, Saccadic Eye Movement, Linear Homeomorphic Model, Neuronal Control Strategy, Oculomotor Plant, Fourier transform, Laplace Transform.

19. ABSTRACT (Continue on reverse if necessary and identify by block number):

System identification techniques were used to estimate muscle forces during horizontal saccadic eye movements in order to better understand the neuronal control strategy. The lateral and medial rectur muscle of each eye was modeled as a parallel combination of an active state tension generator with a viscosity and elastic element, connected to a series elastic element. The eyeball was modeled as a sphere connected to a viscosity and elastic element. The predictions of the model were shown to be in good agreement with the data. The results of extensive analysis did not support the existence of a postulated continuous-time external feedback control mechanism. Analysis of the data, however, did support a time optimal control strategy, a strategy which directs the eyeball to its destination in minimum time for saccades of all sizes.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED UNLIMITED X SAME AS RPT ☐ DTIC USERS ☐	Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Dr. Genevieve Haddad	(202) 767-5021	NL

DD FORM 1473, 83 APR — EDITION OF JAN 73 IS OBSOLETE

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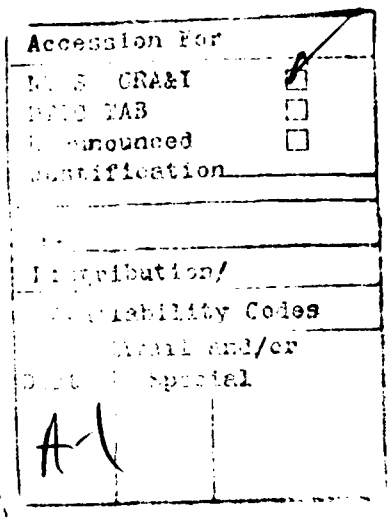
FISCAL REPORT

APPLICATION OF THE SYSTEM IDENTIFICATION TECHNIQUE TO GOAL-DIRECTED SACCADES

Prepared by:	Dr. John D. Enderle
Academic Rank:	Assistant Professor
Department and University:	Department of Electrical Engineering North Dakota State University
Research Location:	Brooks AFB, Clinical Sciences Division, Neurosciences Function
USAF Research Colleague:	Dr. James W. Wolfe
Date:	July 30, 1984
Contract No.:	AFOSR-83-0187

The following expenses were paid from June 1, 1983 to May 31, 1984 by the AFOSR under Grant No. AFOSR-83-0187.

1.	Salaries & Wages	$7,257
2.	Employee Benefits	$ 486
3.	Indirect Costs	$1,177
4.	Equipment	$2,127
	DEC VT100 Terminal	
	Computer Terminal Table & Chair	
	Computer Interface	
5.	Travel	$ 672
6.	Miscellaneous Expenses	$ 281
	Computer Costs	
	Telephone	
	Xeroxing	
	Report Costs	
	Total	$12,000



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FINAL REPORT

APPLICATION OF THE SYSTEM IDENTIFICATION TECHNIQUE TO GOAL-DIRECTED SACCADES

Prepared by:	Dr. John D. Enderle
Academic Rank:	Assistant Professor
Department and University:	Department of Electrical Engineering North Dakota State University
Research Location:	Brooks AFB, Clinical Sciences Division, Neurosciences Function
USAF Research Colleague:	Dr. James W. Wolfe
Date:	July 30, 1984
Contract No.:	AFOSR-83-0187

ABSTRACT

System identification techniques were used to estimate muscle forces during horizontal saccadic eye movements in order to better understand the neuronal control strategy. The lateral and medial rectus muscle of each eye was modeled as a parallel combination of an active state tension generator with a viscosity and elastic element, connected to a series elastic element. The eyeball was modeled as a sphere connected to a viscosity and elastic element. The predictions of the model were shown to be in good agreement with the data. The results of extensive analysis did not support the existence of a postulated continuous-time external feedback control mechanism. Analysis of the data, however, did support a time optimal control strategy, a strategy which directs the eyeball to its destination in minimum time for saccades of all sizes.

I. INTRODUCTION

Saccadic eye movements are among the fastest voluntary muscle movements the human body is capable of producing and are characterized by a rapid shift of gaze from one point of fixation to another. Although the purpose for such an eye movement is obvious, that is, to quickly redirect the eyeball to the target, the neuronal control strategy is not. For instance, does the word "quickly" in the previous sentence imply the most rapid movement possible, or simply a fast as opposed to a slow movement? The present investigation utilizes system identification techniques to estimate muscle forces during horizontal saccadic eye movements in order to better understand the neuronal control strategy.

Until quite recently, models of the saccadic eye movement system involved a ballistic or preprogrammed control to desired eye position based on retinal error alone. (1) - (4) Today, an increasing number of authors are putting forth the idea that goal-directed saccades are controlled by a local feedback loop that continuiously drives the eye to the desired position. This hypothesis, first presented by Vossius in 1960, did not start to gain acceptance until 1975 when Robinson re-examined it. (5)(6) Robinson suggested that saccades originated from neural commands, which encode saccade velocity and duration, to the pulse generator that specifies the desired position of the eye rather than the preprogrammed distance the eye must be moved. The image of the internal representation of the current eye position is subtracted from the internal representation of the desired position to create an error signal that completes the local feedback loop to generate the neural control signal.

This neural pulse continiously drives the eye until the internal representation of the error signal is zero.

One of the basic tasks of this project is to investigate whether an external, continuous-time sensor feedback control mechanism operates during a saccadic eye movement. Studies by other investigators indicate that the profile of a saccade can be altered up to 50 msec after target movement, and that visual information is processed continuously during this internal. (7) (8) Their results indicate that 50 msec after target movement, the saccade profile can not be altered by altering the target location. The present experiment attempts to verify their results by artifically maintaining a constant error throughout the saccade by moving the target the same distance the eyeball moves. This method of artifically openning the feedback loop was used by Collewijn and Van der Mark in their study of the slow phase of optokinetic nystagmus. (9)

A second task in this project involves the investigation of a horizonatal saccadic eye model presented by Enderle. (10) Estimated values of the poles of the transfer function showed an increasing trend as target displacement increased. As stated, a number of reasons may account for this, a significantly nonlinear model and/or an inaccurate model, or poor initial estimates for the parameter values. It is essential that the model for horizontal saccadic eye movements be accurate since it is to be used in sensitively assessing whether an external feedback control mechanism operates during saccades.

II. OBJECTIVES

The main objective of this project is to better understand the mechanisms controlling saccadic eye movements. The specific objectives were:

1. Update and improve the model for horizontal saccadic eye movements.
2. Estimate model parameters using system identification techniques.
3. Investigate the hypothetical operation of an external feedback control mechanism.

III. MODEL OF OCULOMOTOR PLANT

In the development of the model for horizontal saccadic eye movements by Enderle, it was assumed in equation 2 that the time derivative of the antagonist tension was negligibly small. This assumption was a major factor in the resulting functional relationship between target displacement and poles of the transfer function. Another factor effecting this relationship was the initial parameter estimates, which will be discussed in the next section. Using Laplace variable analysis about the operating point or initial eye position, it is possible to derive the following new and more accurate model of horizontal saccadic eye movements appropriate for the system identification technique (See Appendix A1 for details of the derivation and parameter description). (11)

$$\delta(K_{ST}(F_{AG}-F_{ANT})+B_{ANT}\dot{F}_{AG}-B_{AG}\dot{F}_{ANT}) = \dddot{\theta} + P_3\ddot{\theta} + P_2\dot{\theta} + P_1\dot{\theta} + P_0\theta. \quad (1)$$

This model is a modification and correction of the linear homeomorphic model by Bahill et. al. (4) The implementation of this model in the parameter estimation routine resulted in an excellent match between model prediction and

the data. No apparent trends between the poles of the transfer function and target displacement were observed.

Presented in Fig. 1 is a block diagram of the saccadic eye movement system which functions as one element within the overall system controlling all oculomotor movements. The saccadic eye movement system consists of three components; the controller, that is, the agonist and antagonist active state tension generators, the oculomotor plant, and the feedback element H. The oculomotor plant, indicated by the section within the dashed lines in Fig. 2, is based on the Laplace transform of equation 1. Note that under an assumed ballistic control of saccadic eye movements, time delays in the feedback element cause this system to operate in an open loop mode. If, in fact, an external continuous-time feedback control mechanism exists, then openning the feedback loop causes the system to overreact because of the constant error present at the input.

Since no assumptions concerning the saccadic active state tensions are required in the derivation of the modified linear homeomorphic model, any type of control signal may be utilized. For example, investigators may utilize this model when examining alternatives to the pulse-step muscle force. If a continuous-time external feedback control mechanism exists, then a new waveform could be postulated for the active state tension. This waveform could then be revised based on the predicted response of the modified linear homeomorphic model.

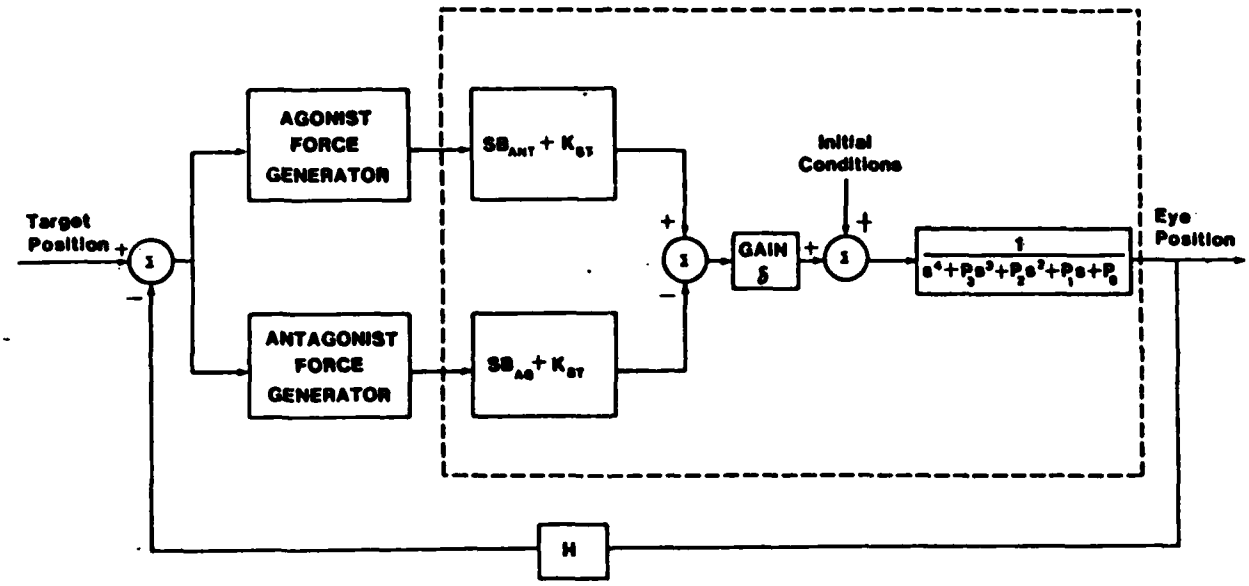

Fig. 1. A block diagram of the oculomotor plant. The section of the diagram within the dashed lines is the modified linear homeomorphic model. The feedback element H is unity.

The mechanical components of the model are conncected such that the rate of change of the muscle forces together with the actual muscle forces drive the eyeball to its final position. Consider the low-pass filtered pulse-step active state tension signals presented in Fig. 2. During the intial pulse phase of the trajectory, the agonist active state tension drives the eyeball and the antagonist active state tension restrains the eyeball. Since the antagonist active state tension falls to zero during this interval, then the rate of change antagonist active state tension, $\dot{F}_{ANT}$, is negative and as indicated by the equation 1, acts to drive the eyeball along with the agonist force, F_{AG}, and the agonist rate of change force, $\dot{F}_{AG}$. As a result, a greater force propels the eyeball than if only the agonist and antagonist active state tensions are used as shown in the plant input curves of Fig. 2. This result is in agreement with the tension data recorded for horizontal saccadic eye movements (12)(13). After the pulse phase of the trajectory, the agonist active state tension falls to a steady state value which results in a negative rate of change agonist active state tension, $\dot{F}_{AG}$. Thus, according to equation 1, the agonist rate of change force, $\dot{F}_{AG}$, acts to restrain the eyeball with the antagonist force, F_{ANT}, and the antagonist rate of change force, $\dot{F}_{ANT}$. As shown for the plant input curves in Fig. 1, the resultant forces act like a dynamic brake which is reduced to zero as the eyeball reaches its destination.

IV. SYSTEM IDENTIFICATION TECHNIQUE

Paramter estimates for the model of the oculomotor system presented in the previous section are found using the system identification technique as described by Enderle. (10)(14) The system identification technique is a frequency response method. Ideally, the transfer function is equal to the

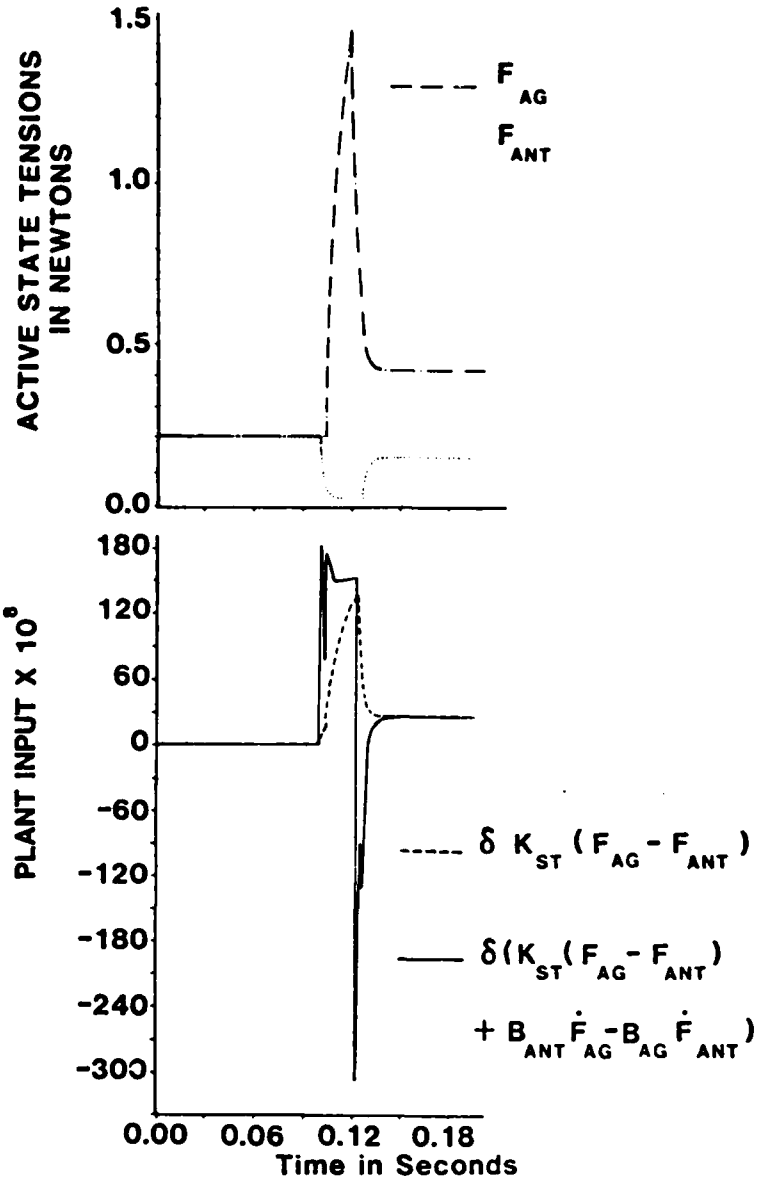

Fig. 2 Transformation of the active state tensions by the numerator terms of the oculomotor plant. One diagram shows the assumed low-pass filtered pulse-step waveform of the agonist and antagonist active state tensions initiated at 100 msec. The antagonist pulse circuiscribes the agonist pulse by 3 msec on each end. Parameter values are from Bahill et al. for simulting a 10 degree horizontal saccadic eye movement (4). The other diagram shows a parital and full transformation of the active state tensions by the numerator terms of the oculomotor plant.

ratio of the Fourier transform of the output to the system input. For the oculomotor system, the transfer function is calculated from the fast eye response to a step target displacement. Because of unequal time delays and eye displacements from saccade to saccade for the same target displacement, it is not possible to use averaging techniques to reduce the effects of measurement noise. Fortunately, the measurement noise is small relative to the input and output signals, so the second method described by Otnes and Enochson is used. (15) The fast eye response measurements are first filtered using a Butterworth low-pass filter with a half-power point at 125 hertz. Transforming the filtered measurements directly by the Fast Fourier algorithm resulted in distortion due to truncation, since the signal did not go to zero at steady state. This is circumvented by subtracting the steady state value from each sample, passing this signal through a Kaiser window, packing with zero's, and Fast Fourier transforming the modified signal. The Fourier transform of the fast eye response is equal to the Fast Fourier transform of the modified signal plus the Fourier transform of the unit step with amplitude equal to the steady state value. The input signal is the Fourier transform of the unit step function with amplitude equal to the steady state value. Parameter estimates for the oculomotor model are calculated using the conjugate gradient search program similar to Seidel's, which minimizes the integral of the absolute value of the error squared between the model and the data. (16)

Data were collected from subjects seated before a target display of seven small red light emitting diodes (LED), each separated by five degrees. The subject's head was restrained by a bit-bar. The subject was instructed to

follow the "jumping" target which moved from the center position to any of the other LED's and then returned to the center position. The order of the target positions as well as the time interval between displacements were randomized. Data were only recorded for the initial displacement from the center position.

Horizontal eye movements were recorded from each eye using an infrared signal reflected from the anterior surface of the cornea-scleral interface with instrumentation described by Engelken et. al. (17). Signals for both eyes tracking were digitized using the analog/digital converter of the DECLAB PDP 11/34 computer and stored in disk memory. These signals were sampled at a rate of 1000 samples per second for one-half second.

Presented in Fig. 1 is a block diagram of the oculomotor system, which is suitable for the case of a ballistic controller, or as indicated, with an external continuous-time feedback control configuration. Under the assumption of ballistic control, the system operates in an open loop mode due to time delays in the feedback element. Under the assumption of an external continious-time controller during experiments in which the feedback loop is artifically openned, the system also operates in an open loop mode. Thus, the analysis of the data is performed with an open loop system regardless of the type of control assumed operating. However, the signals describing the agonist and antagonist force generators are quite different dependent on the type of controller.

Under an assumed ballistic control mechanism, the agonist and antagonist active state tensions are described with the following low-pass filtered pulse-step waveforms

$$F_{AG}(t) = F_{AGO}u(t) + (F_{AGP}-F_{AGO})(1-\exp\{-(t-t_p)/\tau_{ac}\})u(t-t_p) - (F_{AGP}-F_{AGS})(1-\exp\{-(t-t_p-t_d)/\tau_{de}\})u(t-t_p-t_d),$$

$$F_{ANT}(t) = F_{ANTO}\,u(t) - F_{ANTO}(1-\exp\{-(t-t_p)/\tau_{de}\})u(t-t_p) + F_{ANTS}(1-\exp\{-(t-t_p-t_d)/\tau_{de}\})u(t-t_p-t_d),$$

where

F_{AGO} = initial magnitude of the agonist active state tension,

F_{AGP} = pulse magnitude of the agonist active state tension,

F_{AGS} = step magnitude of the agonist active state tension,

F_{ANTO} = initial magnitude of the antagonist active state tension,

F_{ANTS} = step magitude of the antagonist active state tension,

t_p = latent period, the time interval between the target movement and the start of the eye movement,

t_d = duration of the agonist pulse active state tension,

τ_{ac} = activation time constant,

and

τ_{de} = deactivation time constant.

The steady state antagonist active state tension terms, F_{ANTO} and F_{ANTS}, are removed from the analysis since

$$F_{ANTO} = F_{AGO} - P_o\,\theta(0)/(\delta\, K_{SE}K_{ST})$$

and

$$F_{ANTS} = F_{AGS} - P_o\theta(\infty)/(\delta\, K_{SE}K_{ST})$$

according to equations 7 and 8 in Appendix A1.

Physiological data was used to estimate all of the initial parameter estimates. (4)(13)(18) Specifically

$K = 66.4$ Nt/m (Robinson)

$K_{LT} = 32.$ Nt/m (Robinson)

$K_{SE} = 125$ Nt/m (Collins)

$B_{AG} = 3.5$ Nt sec/m (Bahill)

$B_{ANT} = 1.66$ Nt sec/m (Bahill)

$B = 3.1$ Nt sec/m (Collins)

$J = 2.2 \times 10^{-3}$ Nt sec^2/m (Robinson)

The value of K is determined from the steady state agonist and antagonist muscle tensions

$$T_{AG} - T_{ANT} = Kx$$

where

$$T_{AG} = 17 + \theta \text{ gm tension} \quad (2)$$

and

$$T_{ANT} = 17 - 0.3\,\theta \text{ gm tension} \quad (3)$$

from Fig. 1 of Robinson et. al. (18) The value of the muscle viscosity terms reported by Bahill et. al. are modified according to their incorrectly derived result. (4)

and

$$B_{AG} = K_{ST} B_{AG} / K_{SE}$$
$$B_{ANT} = K_{ST} B_{ANT} / K_{SE}$$

Substituting these values into the expressions for δ, P_0, P_1, P_2 and P_3 found in Appendix A1, yields

$$\delta = 5.0938109 \times 10^7$$

$$P_3 = 1.548 \times 10^3$$

$$P_2 = 3.4455 \times 10^5$$

$$P_1 = 1.9724 \times 10^7$$

$$P_0 = 2.2631 \times 10^8$$

The initial extimates for the four real eigenvalues as determined from the characteristic equations for this system are -15.27, -66.1, -173.36, and -1293.28.

Now, the steady state active state tensions, F_{AGO}, F_{AGS}, F_{ANTS}, F_{ANTO}, are computed from

$$F_{AG} = \frac{KST}{K_{SE}} T_{AG} + K_{LT}(x - x_{P1})$$

and

$$F_{ANT} = \frac{KST}{K_{SE}} T_{ANT} - K_{LT}(x + x_{P1}),$$

derived from Fig. 1 of Appendix A1. Substituting equations 2 and 3 into the above equations with x_{P1} equal to 3.14 mm (assumed from Robinson) yields

$$F_{AG} = .1089 + .01850 \quad NT$$

and

$$F_{ANT} = .1089 - 9.838 \times 10^{-3}\theta \quad NT.$$

Note that when F_{ANT} at steady state is greated than 11°, then F_{ANT} equals 0. The initial estimate of the pulse magnitude of the active state tension, F_{AGP}, is 0.9806 (a maximum innernation signal of 100 gms). (13)

The initial estimate of the duration of the agonist pulse active state tension is estimated directly from the data. The maximum velocity is computed using a two-point central difference derivate method. (19) The initial

estimate of t_p is then the time interval between the start of the saccade and the time at maximum velocity.

The initial estimate of the deactivation time constant is also estimated directly from the data as the time interval between maximum velocity and the end of the saccade divided by 4. The activation time constant equals 2 msec. (20)

Great care was exercised in evaluating initial parameter estimates since large differences from the true values could cause the estimation routine to converge to suboptimal and nonphysiologically consistent results. The precision of the parameter estimation routine results presented in Figures 3 and 4 for a 10^{o} target movement are typical for the three subjects tested. There was no significant trend in the values of the eigenvalues with target displacement and the estimates correlated well with their physiological data derived values. Listed in Table 1 are the final estimated values for the eigenvalues for target displacements of 5, 10, and 15 degrees.

V. EXTERNAL FEEDBACK CONTROL MECHANISM

Experiments were conducted in which the hypothesized continuous-time external feedback control mechanism was artifically openned. Data was collected on three subjects in two stages. First stage data consisted of natural saccades, elicited from target displacements of -15 to 15 degrees. The second stage consisted of data collected by artifically maintaining a constant error of 5^{o} throughout the saccade by moving the target (under computer control) the same distance the eyeball moved. This type of movement will be referred to as the modified target trajectory. The experiment started by the target moving $\pm$ 5 degrees. The subject responded by following the

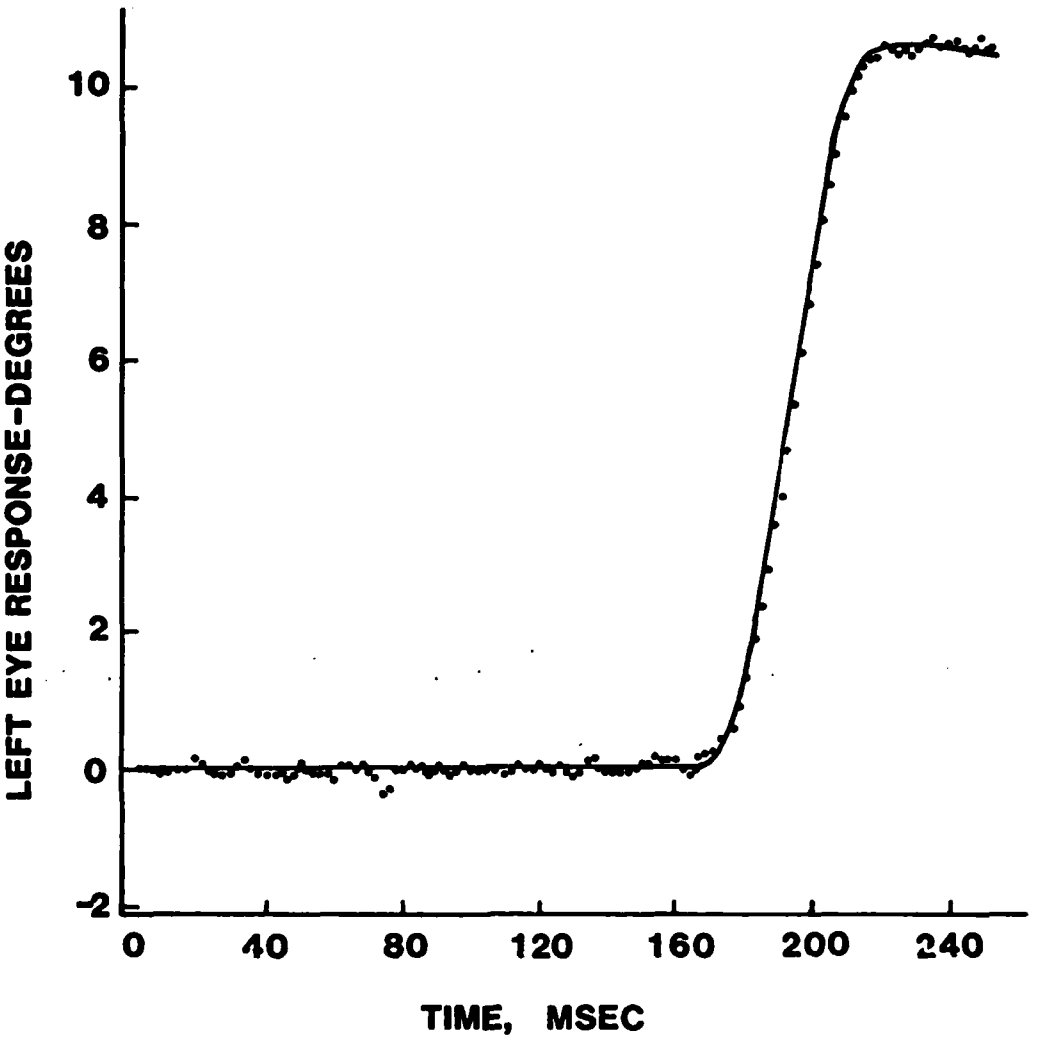

Fig. 3. Time response for a 10^{o} saccadic eye movement.

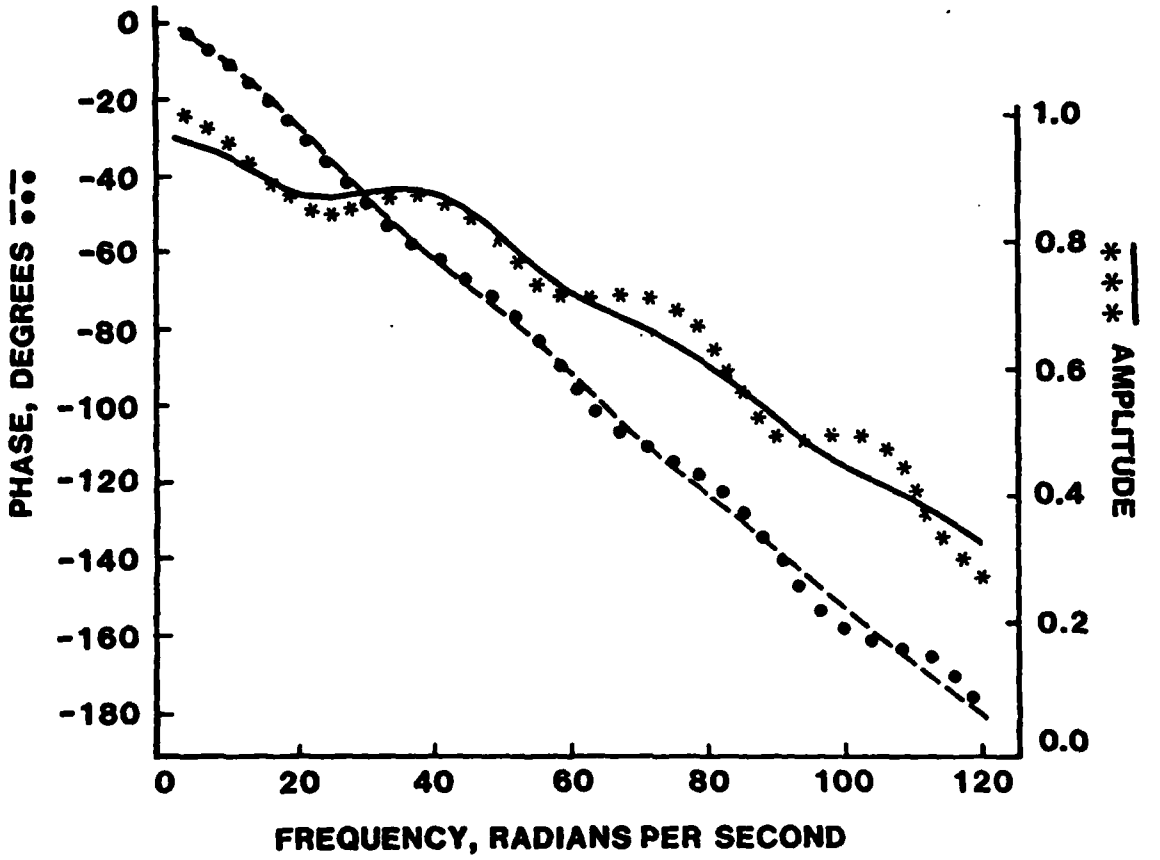

Fig. 4. Frequency response for a 10° saccadic eye movement.

TABLE 1

Final parameter estimates for the four real eigenvalues for target displacements of 5, 10, and 15 degrees left of center position.

		TARGET DISPLACEMENTS		
		5^{o}	10^{o}	15^{o}
LEFT EYE	Eigenvalue 1	-14.4	-13.9	-13.7
	Eigenvalue 2	-83.8	-84.4	-93.6
	Eigenvalue 3	-190.9	-174.6	-157.4
	Eigenvalue 4	-1298.7	-1283.2	-1259.4
RIGHT EYE	Eigenvalue 1	-14.8	-14.4	-12.35
	Eigenvalue 2	-80.8	-84.9	-88.2
	Eigenvalue 3	-185.8	-165.4	-160.1
	Eigenvalue 4	-1302.2	-1261.6	-1223.7

jumping target. Instantaneously, as the eyeball moved, the target moved that same distance. Significant differences in saccade profile between the two stages would indicate the possible existence of an external feedback mechanism. Data was first analyzed using the two-point central difference method, and then using the system identification technique.

The results of the analysis of the data on the three subjects using the two-point central difference method is contained in Appendix A2. (19) The results of the analysis did not support the postulated continuous-time external feedback control mechanism. The time internal from the start of the saccade to the time at peak velocity for subjects 2 and 3 did show marked variation ranging from 21 msec to 8 msec, however. Most of the values were clustered about evenly into two grouping 16 msec and 10 msec. These time interval results are plotted in Fig. 5. It is interesting to note that the 5^{o} movements elicited under natural conditions were always in the larger time interval grouping of 16 msec for each of the three subjects. It is speculated that if there were a continuous-time external feedback control mechanism, then both the time interval and the final eye displacement would have been much larger than under normal conditions.

The results of the analysis of the data on the three subjects using the system identification technique confirmed the previous findings. Additionally, estimates of the pulse magnitude of the agonist active state tension F_{AGP} were clustered about evenly into two groupings, corresponding to the previous groupings, 0.95 NTS. and 0.85 NTS. These results are plotted in Fig. 6A. The estimated pulse magnitude for the agonist active state tension as a function of displacement for the natural saccades are illustrated in Fig.

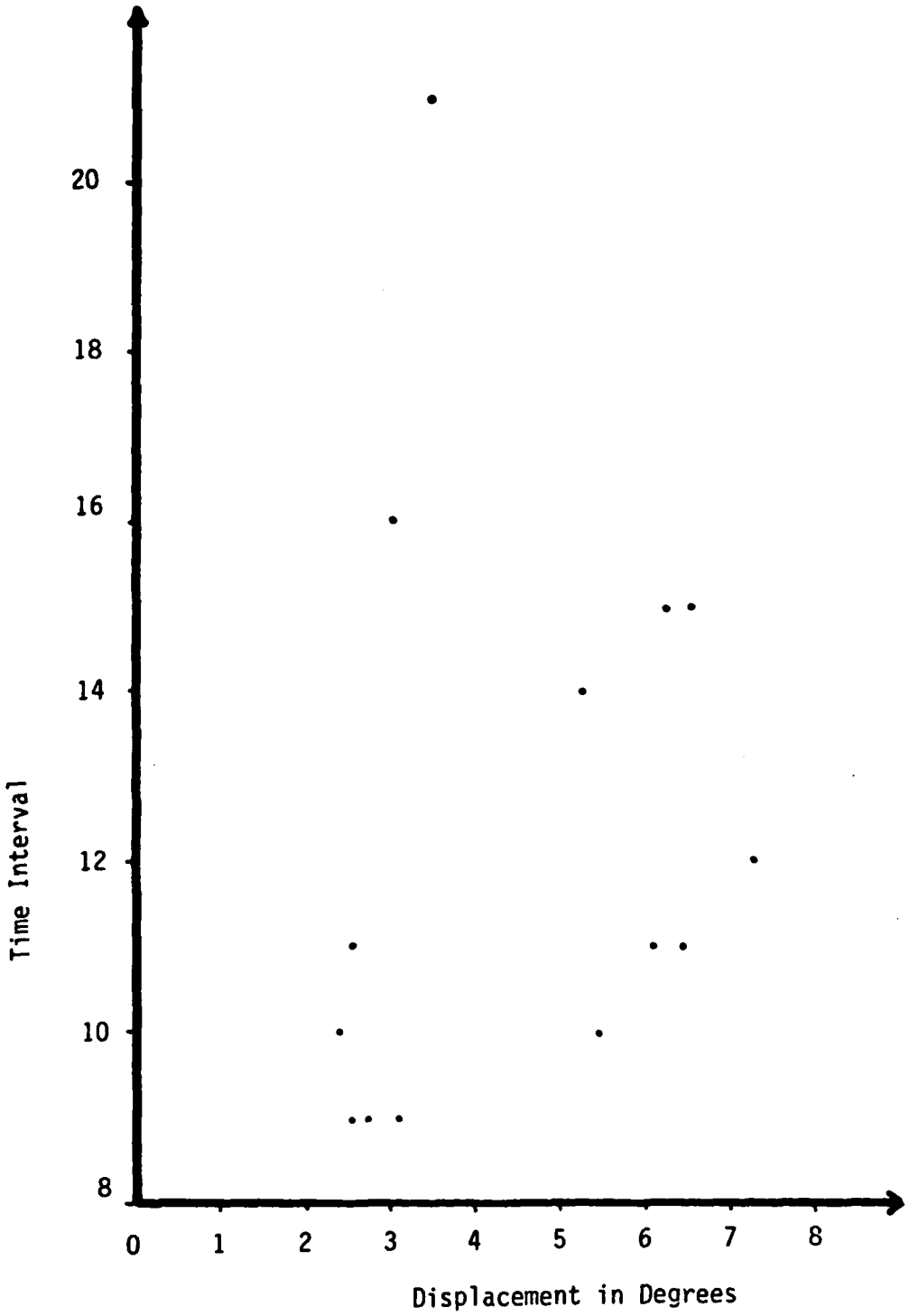

Fig. 5. The time interval from the start of the saccade to the the time at peak velocity vs absolute value of displacement for subject 2.

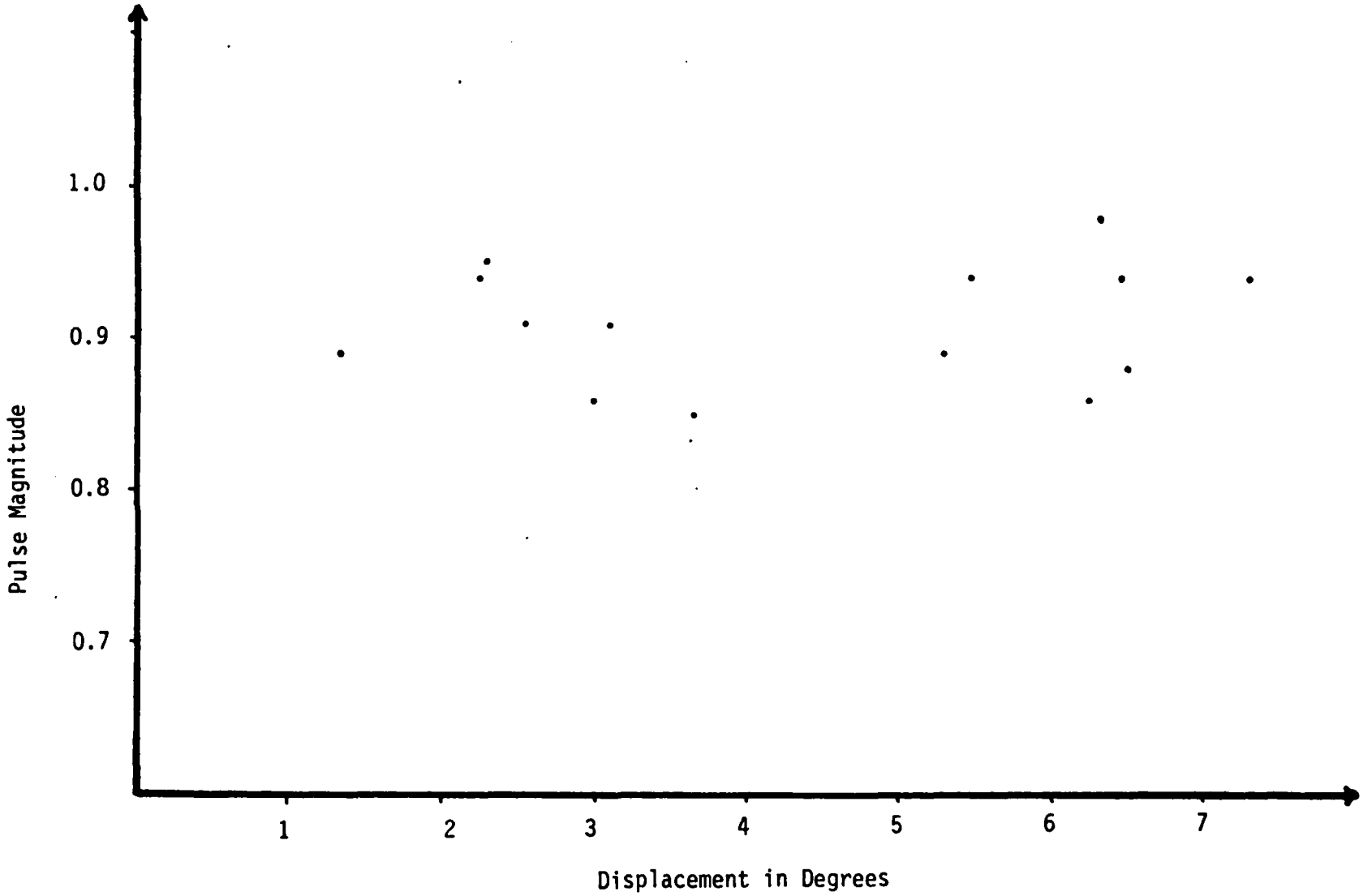

Fig. 6A. The estimated values of the pulse magnitude of the agonist active state tension vs absolute value of displacement for subject 3 during the stage 2 experiment.

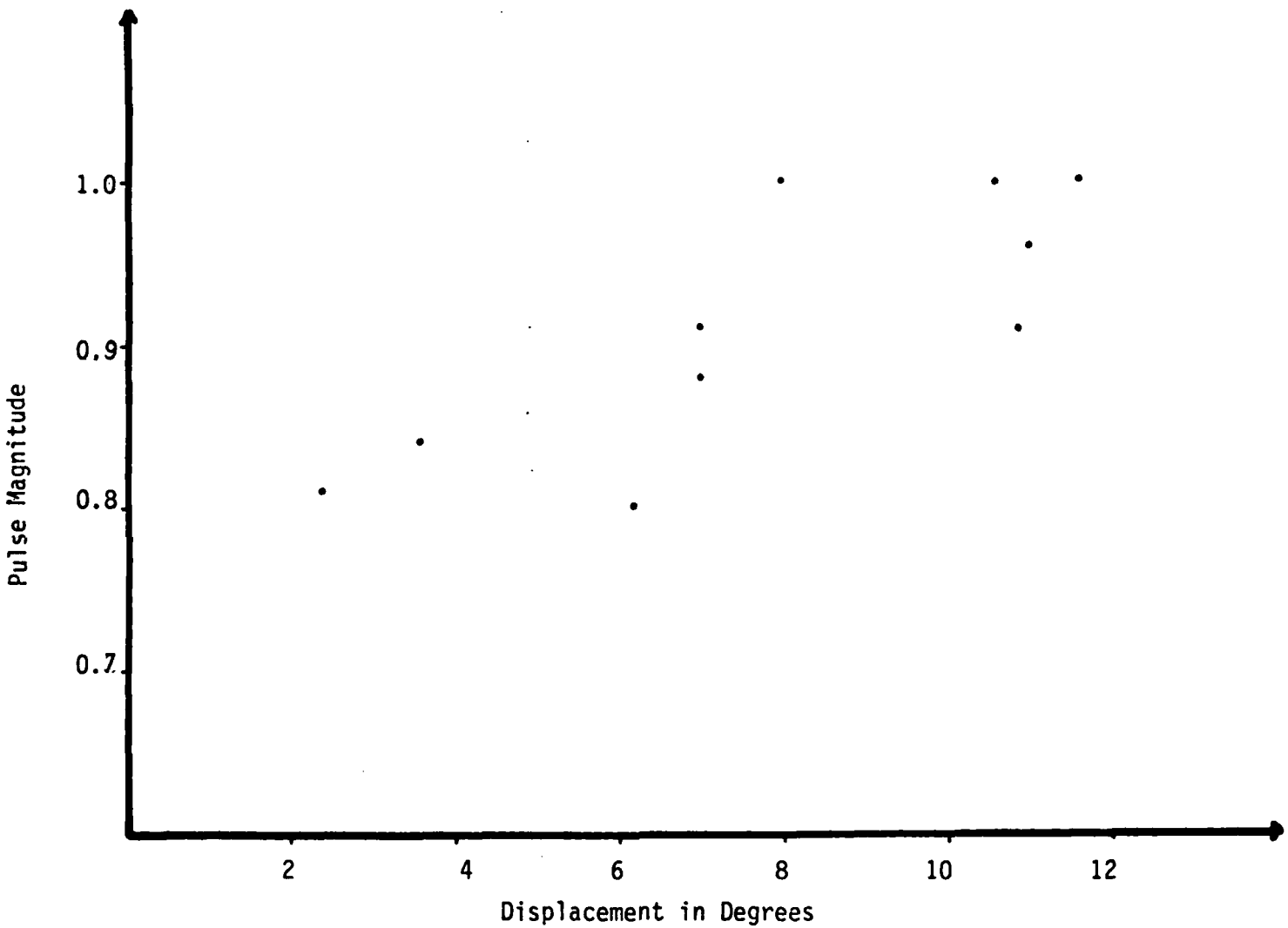

Fig. 6B. The estimated values of the pulse magnitude of the agonist active state tension vs. absolute value of displacement for subject 3 during the stage 1 experiment.

6B. One pronounced feature evident from these two graphs is that there does not appear to be a strong relationship between pulse magnitude and displacement. In fact the variation in F_{AGP} seen in Fig. 6A is approximately the same size variation seen in Fig. 6B. Apparently the magnitude of the agonist pulse is maximum regardless of the amplitude of the saccade, and only the duration of the agonist pulse affects the size of the saccade. Under these conditions, the eyeball is driven to its destination in minimim time for saccades of all sizes. (See Appendix A3 for a more complete discussion of this time optimal model). (21)

VI. RECOMMENTATIONS

The results obtained during the performance of this project indicate that the saccadic eye movement system operates under a time optimal control strategy without continuous-time external feedback sensory information. The implication that the eyeball is directed to its desination in minimum time for saccades of all sizes certainly deserves additional attention considering the small number of subjects tested. This new hypothesis certainly should be investigated further in light of the current theory on the neuronal control of saccadic eye movements. Particular attention should be paid to the large variation seen with saccades of the same size. Furthermore, the importance of the second phase of the saccade, that is, when the agonist pulse is falling to its steady state value, must be understood in relation to the time optimal approach.

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14. J.D. Enderle, "Estimation of Saccadic Eye Movement Muscle Forces using System Identification Techniques," Second Southern Biomedical Engineering Conference, San Antonio, Texas, September 1983: 267-270.

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APPENDIX

A1. Copy of an artical to be published in IEEE Transaction on Biomedical Engineering in October 1984.

A2. Summary of Data Analysis

A3. Conference Proceedings Publications by Investigator Sponsored by AFOSR.

1. Enderle, J.D., Wolfe, J.W., and Yates, J.T., Optimal Control of Saccadic Eye Movements. Twenty-First Annual Rocky Mountain Bioengineering Symposium, Boulder, Colorado, April 1984: 143-148.

2. Enderle, J.D., Wolfe, J.W., and Yates, J.T., A Model of Horizontal Saccadic Eye Movements. Seventeenth Hawaii International Conference on System Sciences, Honolulu, Hawaii, January 1984: Vol. II 337-334.

3. Enderle, J.D., Estimation of Saccadic Eye Movement Muscle Forces Using System Identification Techniques. Second Southern Biomedical Engineering Conference, San Antonio, Texas, September 1983: 267-270.

THE LINEAR HOMEOMORPHIC SACCADIC EYE MOVEMENT MODEL -- A MODIFICATION

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ABSTRACT

The objective of this study was the modification of a linear homeomorphic horizontal saccadic eye movement model to a direct programming state-space representation through Laplace variable analysis about the operating point or initial eye position. The lateral and medial rectus muscle of each eye is modeled as a parallel combination of an active state tension generator with a viscosity and elastic element, connected to a series elastic element. The eyeball is modeled as a sphere connected to a viscosity and elastic element. Each of these elements is assumed to be ideal and linear.

INTRODUCTION

Models of oculomotor function are important in the development of clinically useful diagnostic tools and in understanding the neurophysiology of eye movements [1] - [8]. The complexity of these models and their improved correlation with physiological data has increased since Westheimer first presented a model of saccadic eye movements in 1954 [9]. Recently, a sixth-order linear homeomorphic horizontal saccadic eye movement model was developed that provided an excellent match between the model predictions and the data [10]. However, the complexity of the model has hampered its widespread use in clinical studies of oculomotor function. Investigators have utilized zero-order or second-order oculomotor plant functions with reduced precision and little or no theoretical considerations [1] - [3] [5] [6] [11]. This paper presents a modification of the linear homeomorphic saccadic eye movement model. The simplicity of the derived model makes it ideal for use in the development of more sensitive tests of oculomotor pathology and in the description of normal oculomotor function.

MODEL OF OCULOMOTOR PLANT

Figure 1 illustrates the mechanical components of the oculomotor plant for horizontal eye movements, the lateral and medial rectus muscle, and the eyeball. The agonist muscle is modeled as a parallel combination of an active state generator

F_{AG}, viscosity element B_{AG} and elastic element K_{LT}, connected to a series elastic element K_{SE}. The antagonist muscle is similarly modeled as a parallel combination of an active state generator F_{ANT}, viscosity element B_{ANT} and elastic element K_{LT}, connected to a series elastic element K_{SE}. The eyeball is modeled as a sphere with moment of inertia J_p connected to viscosity element B_p and elastic element K_p. The passive elasticity for each muscle is included in spring K_p for ease in analysis. Each of the elements defined in the model of the oculomotor plant is ideal and linear.

The model presented in Fig. 1 is consistent with the linear homeomorphic model except for the treatment of initial conditions. The muscles are shown to be extended from equilibrium, a position of rest, at the primary position (looking straight ahead) consistent with physiological evidence [12]. The average length of the rectus muscle at the primary position is approximately 40 mm, and at the equilibrium position is approximately 37 mm [12]. θ is the angle the eyeball is deviated from the primary position, and variable x is the length of the arc traversed. When the eye is at the primary position, both θ and x are equal to zero. Variables x_1 through x_4 describe the displacement from equilibrium for the stiffness elements in each muscle. Values x_{p1} through x_{p4} are the displacements from equalibrium for each of the variables x_1 through x_4 at the primary position. The total extension of the muscle from equilibrium at the primary position is x_{p1} plus x_{p2} or x_{p3} plus x_{p4} which equals approximately 3mm [12]. It is assumed that the lateral

and medial rectus muscles are identical, such that x_{p1} equals x_{p4} and x_{p3} equals x_{p2}. The following relationships (evident from Fig. 1) exist among variables x_1, x_4, θ, and x.

$$x_1 = x - x_{p1}, \tag{1}$$

$$x_4 = x + x_{p4}, \tag{2}$$

and

$$\theta = 57.296\ x/r, \tag{3}$$

where r equals the radius of the eyeball with a value of approximately 11 mm. The initial condition is assigned to variables x_1, x_4, and θ through x; that is, variable x at time 0, $x(0)$, and equations 1 through 3. Initial conditions are also assigned to variables x_2 and x_3 as $x_2(0)$ and $x_3(0)$, and to the agonist and antagonist active state tensions as $F_{AG}(0)$ and $F_{ANT}(0)$.

A set of three equations is written to describe the oculomotor plant by summing the forces acting at junctions 2 and 3, the equilibrium positions for x_2 and x_3, and the torques acting on the eyeball.

$$F_{AG} = K_{LT}x_2 + B_{AG}\dot{x}_2 + K_{SE}(x_2 - x_1), \tag{4}$$

$$F_{ANT} = K_{SE}(x_4 - x_3) - K_{LT}x_3 - B_{ANT}\dot{x}_3, \tag{5}$$

and

$$rK_{SE}(x_2 + x_3 - x_1 - x_4) = J_p\ddot{\theta} + B_p\dot{\theta} + K_p\theta. \tag{6}$$

Equations 4 and 5 differ from those of Bahill *et al.* with regard to the viscosity terms, a constant $K_{SE}/(K_{LT} + K_{SE})$ is missing in the viscosity term in their equations 14 and 15 [10]. Equations 4 through 6 could be simplified and written as one equation by taking the Laplace transform of each equation and eliminating all variables except x. However, the initial conditions must be

considered in the transform. An alternative method is to reduce the equations to include only changes from the initial eye position.

Introduce

$$\hat{x} = x - x(0),$$

$$\hat{\theta} = \theta - \theta(0),$$

$$\hat{x}_1 = x_1 - x_1(0),$$

$$\hat{x}_2 = x_2 - x_2(0),$$

$$\hat{x}_3 = x_3 - x_3(0),$$

$$\hat{x}_4 = x_4 - x_4(0),$$

$$\hat{F}_{AG} = F_{AG} - F_{AG}(0),$$

and

$$\hat{F}_{ANT} = F_{ANT} - F_{ANT}(0).$$

Equations 4 through 6 are rewritten in terms of changes from the initial eye position as

$$\hat{F}_{AG} = K_{ST}\,\hat{x}_2 + B_{AG}\,\dot{\hat{x}}_2 - K_{SE}\hat{x}, \tag{7}$$

$$\hat{F}_{ANT} = K_{SE}\,\hat{x} - K_{ST}\,\hat{x}_3 - B_{ANT}\,\dot{\hat{x}}_3, \tag{8}$$

and

$$K_{SE}(\hat{x}_2 + \hat{x}_3 - 2\hat{x}) = J\ddot{\hat{x}} + B\dot{\hat{x}} + K\hat{x}, \tag{9}$$

where

$$K_{ST} = K_{SE} + K_{LT},$$

$$J = 57.296\ J_p/r^2,$$

$$B = 57.296\ B_p/r^2,$$

and $$K = 57.296\ K_p/r^2.$$

Note that variables $\hat{x}_1$ and $\hat{x}_4$ are eliminated from the above equations since they are both equal to x, and variable $\hat{\theta}$ through equation 3.

Equations 7 through 9 are written as one equation by taking the Laplace transform of each equation and eliminating variables $\hat{x}_2$ and $\hat{x}_3$ yielding

$$K_{SE}(SB_{ANT} + K_{ST})L\{\hat{F}_{AG}\} - K_{SE}(SB_{AG} + K_{ST})L\{\hat{F}_{ANT}\} =$$

$$(C_4S^4 + C_3S^3 + C_2S^2 + C_1S + C_0)\ \hat{X}(S), \qquad (10)$$

where $C_4 = JB_{ANT}B_{AG}$,

$C_3 = JK_{ST}(B_{AG} + B_{ANT}) + BB_{ANT}B_{AG}$,

$C_2 = JK_{ST}^2 + BK_{ST}(B_{AG} + B_{ANT}) + B_{AG}B_{ANT}(K + 2K_{SE})$,

$C_1 = BK_{ST}^2 + (B_{AG} + B_{ANT})(KK_{ST} + 2K_{SE}K_{ST} - K_{SE}^2)$,

and $C_0 = KK_{ST}^2 + 2K_{SE}K_{ST}K_{LT}$.

Transforming equation 10 back into the time domain yields

$$K_{SE}(K_{ST}(\hat{F}_{AG} - \hat{F}_{ANT}) + B_{ANT}\dot{\hat{F}}_{AG} - B_{AG}\dot{\hat{F}}_{ANT}) = C_4\ddot{\hat{x}} + C_3\dddot{\hat{x}} + C_2\ddot{\hat{x}} + C_1\dot{\hat{x}} + C_0\hat{x}. \qquad (11)$$

This equation describes the trajectory of the eyeball during a horizontal saccadic movement from the initial eye position. By introducing the initial eye position and active state tension, and noting that

$$\dot{F}_{AG} = \dot{\hat{F}}_{AG},$$

and $$\dot{F}_{ANT} = \dot{\bar{F}}_{ANT},$$

we have

$$K_{SE}(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}\dot{F}_{AG} - B_{AG}\dot{F}_{ANT}) = C_4\dddot{x} + C_3\ddot{x} + C_2\dot{x} + C_1\dot{x} + C_0x. \quad (12)$$

Since data are available for the angle of the eyeball with respect to the primary position, the angle θ is used instead of x which yields

$$\delta(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}\dot{F}_{AG} - B_{AG}\dot{F}_{ANT}) = \dddot{\theta} + P_3\ddot{\theta} + P_2\dot{\theta} + P_1\dot{\theta} + P_0\theta, \quad (13)$$

where

$$\delta = 57.296K_{SE}/rC_4,$$

$$P_3 = C_3/C_4$$

$$P_2 = C_2/C_4,$$

$$P_1 = C_1/C_4,$$

and $$P_0 = C_0/C_4.$$

State equations are usually written for ease in simulating saccadic eye movements with programs available for the digital computer such as the Continuous System Modeling Program. One set of state equations is given below in terms of state variables θ_1 through θ_4, where

$\theta_1 = \theta =$ angular position,

$\theta_2 = \dot{\theta}_1 =$ angular velocity,

$\theta_3 = \dot{\theta}_2 =$ angular acceleration,

and $\theta_4 = \dot{\theta}_3 =$ angular jerk,

with state equations

$$\dot{\theta}_1 = \theta_2,$$

$$\dot{\theta}_2 = \theta_3,$$

$$\dot{\theta}_3 = \theta_4,$$

and

$$\dot{\theta}_4 = -P_0\theta_1 - P_1\theta_2 - P_2\theta_3 - P_3\theta_4 + (K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}\dot{F}_{AG} - B_{AG}\dot{F}_{ANT}).$$

The agonist and antagonist active state tension terms are not included as state variables since they do not describe the energy state of the system.

DISCUSSION

The objective of this study was to modify the linear homeomorphic model to better understand the complex mechanisms underlying saccadic eye movements, and to derive a simpler representation of the model in an effort to increase its potential value in evaluating normal and pathological oculomotor function. We feel we have accomplished this objective as illustrated in the previous section. Since no additional assumptions are introduced, the excellent match between model prediction and the data reported by Bahill et al. remains intact [10]. The elimination of variables x_2 and x_3 from the linear homeomorphic model leads to a simpler direct programming state-space representation. Note that the state variables, velocity, acceleration and jerk, are estimable from the measured position of the eyeball either from analog electronic processing or through computational techniques such as the two point central difference method [13] [14]. Human physiological data is available for estimating each parameter in the linear homeomorphic model [12] [15]. While the

tension produced by each muscle is experimentally known [15], the actual active state tensions (muscle forces), F_{AG} and F_{ANT}, and the initial conditions on variables x_1 through x_4 are unknown. Since the initial conditions for variables x_1 through x_4 are not needed in the modified linear homeomorphic model, only the saccadic muscle forces need be specified. One typically assumes an input waveform for the agonist and antagonist active state tensions that is a simple function of the observed motoneuronal signal [16][17][18].

Presented in Fig. 2 is a block diagram of the saccadic eye movement system which functions as one element within the overall system controlling all oculomotor movements. The saccadic eye movement system consists of three components; the controller, that is, the agonist and antagonist active state tension, the oculomotor plant, and the feedback element H. Since no assumptions concerning the saccadic active state tensions are required in the derivation of the modified linear homeomorphic model, any type of control signal may be utilized. For example, investigators may utilize this model when examining alternatives to the pulse-step muscle force. The oculomotor plant, indicated by the section within the dashed lines in Fig. 2, is based on equation 10. The four real eigenvalues as determined from the characteristic equation for the parameter values given by Bahill et al. are -17, -75, -171, and -1307 [10].

The mechanical components of the model are connected such that the rate of change of the muscle forces together with the actual muscle forces drive the eyeball to its final position.

Consider the low-pass filtered pulse-step active state tension signals presented in Fig. 3. During the initial pulse phase of the trajectory, the agonist active state tension drives the eyeball and the antagonist active state tension restrains the eyeball. Since the antagonist active state tension falls to zero during this interval, then the rate of change antagonist active state tension, $\dot{F}_{ANT}$, is negative and as indicated by the equation 13, acts to drive the eyeball along with the agonist force, F_{AG}, and the agonist rate of change force, $\dot{F}_{AG}$. As a result, a greater force propels the eyeball than if only the agonist and antagonist active state tensions are used as shown in the plant input curves of Fig. 3. This result is in agreement with the tension data recorded for horizontal saccadic eye movements [15][19]. After the pulse phase of the trajectory, the agonist active state tension falls to a steady state value which results in a negative rate of change agonist active state tension, $\dot{F}_{AG}$. Thus, according to equation 13, the agonist rate of change force, $\dot{F}_{AG}$, acts to restrain the eyeball with the antagonist force, F_{ANT}, and the antagonist rate of change force, $\dot{F}_{ANT}$. As shown for the plant input curves in Fig. 3, the resultant forces act like a dynamic brake which is reduced to zero as the eyeball reaches its destination.

In summary, a modification of a linear homeomorphic model for horizontal saccadic eye movements is presented which allows one to estimate or directly measure relevant parameters and initial conditions from physiological data. Since no assumptions concerning the saccadic active state tensions are used in the

derivation of the model, any type of control input can be used in evaluating the saccadic system.

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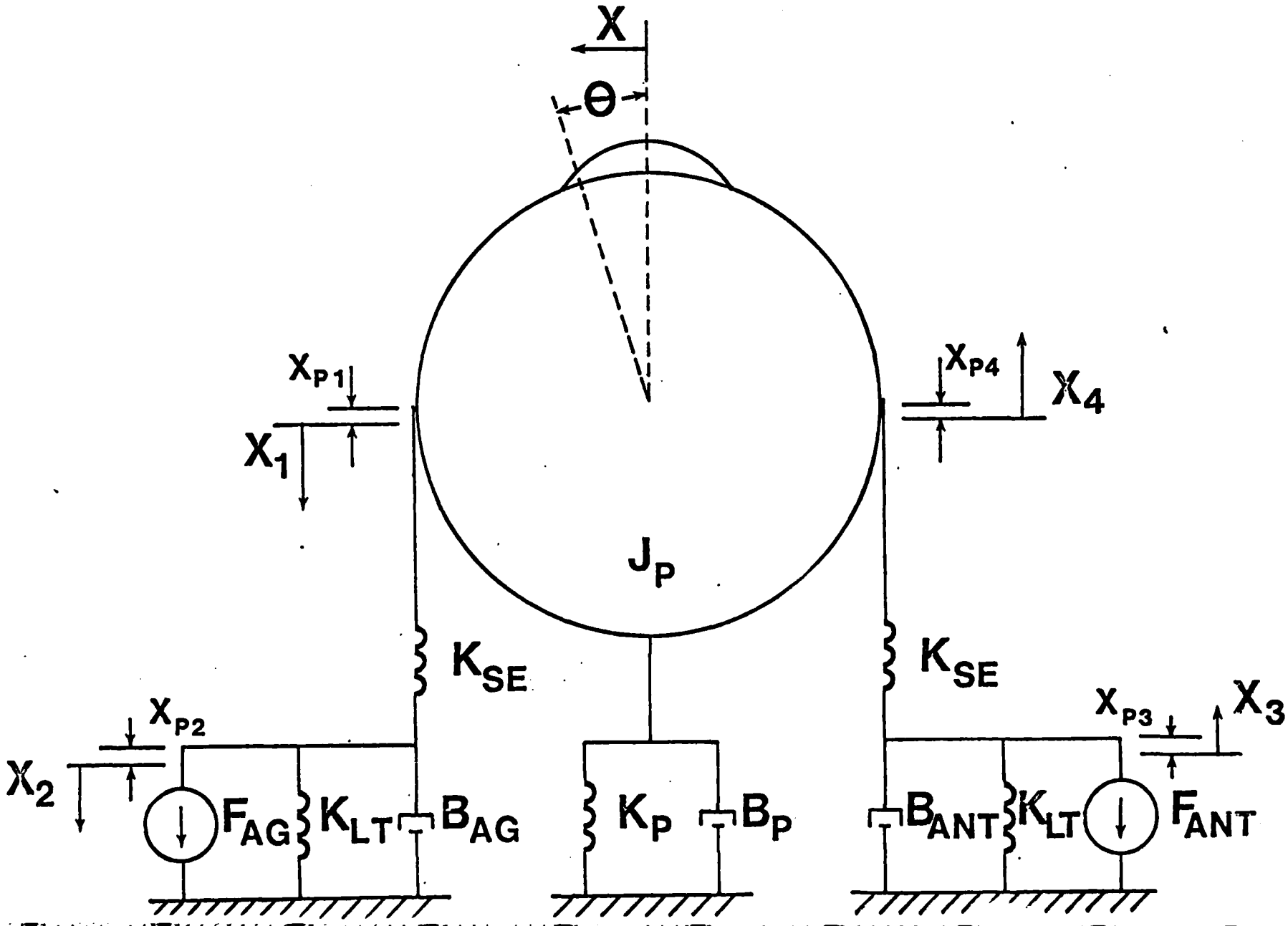
Fig. 1. The mechanical components for the oculomotor plant at the primary position.

Fig. 2. A block diagram of the oculomotor plant. The section of the diagram within the dashed lines is the modified linear homeomorphic model. The feedback element H is unity.

Fig. 3. Transformation of the active state tensions by the numerator terms of the oculomtor plant. One diagram shows the assumed low-pass filtered pulse-step waveform of the agonist and antagonist active state tensions initiated at 100 msec. The antagonist pulse circumscribes the agonist pulse by 3 msec on each end. Parameter values are from Bahill *et al.* for simulating a 10 degree horizontal saccadic eye movement [10]. The other diagram shows a partial and full transformation of the active state tensions by the numerator terms of the oculomotor plant.


X
Θ
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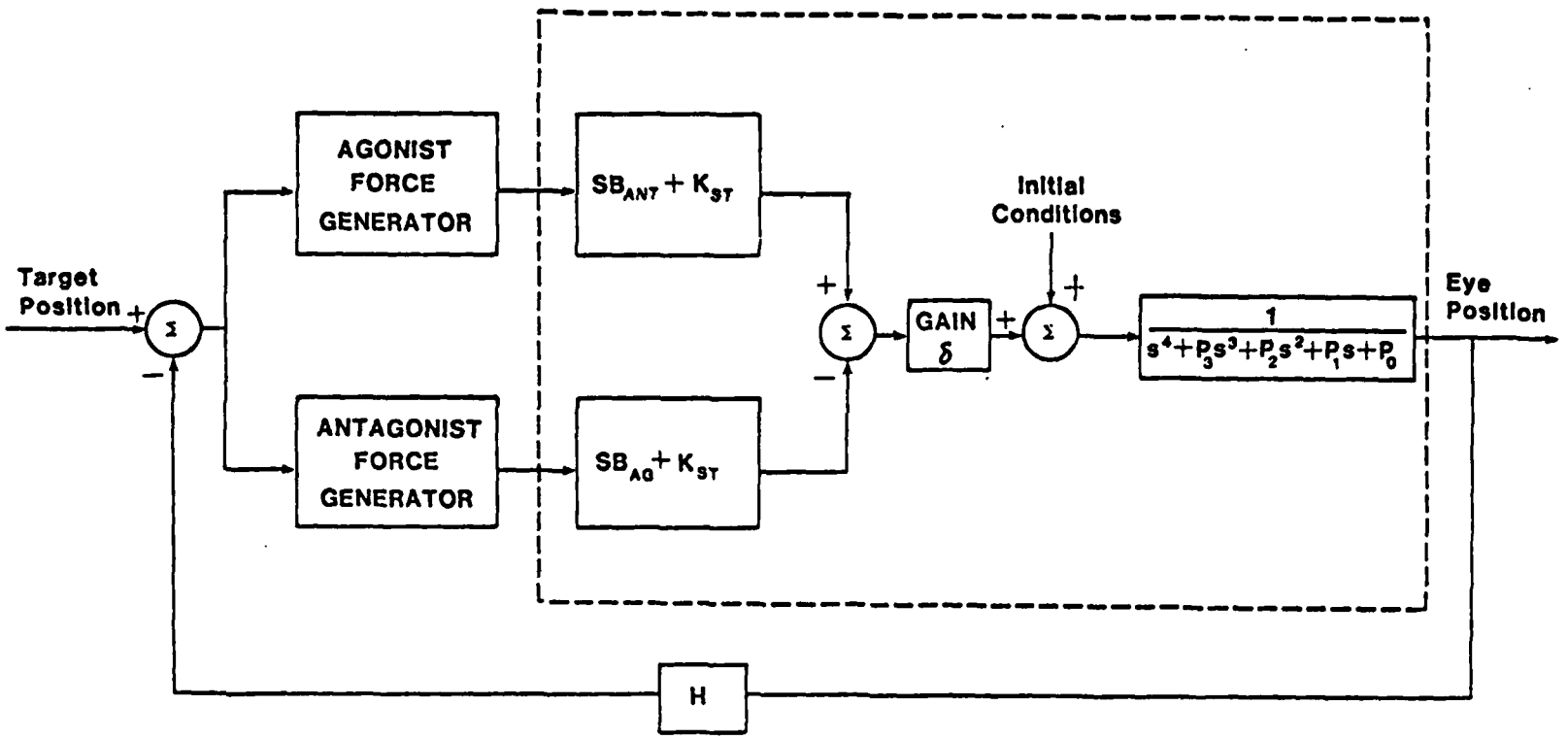
AGONIST FORCE GENERATOR
$SB_{ANT} + K_{ST}$
Initial Conditions
Target Position
+
Σ
−
+
Σ
−
GAIN δ
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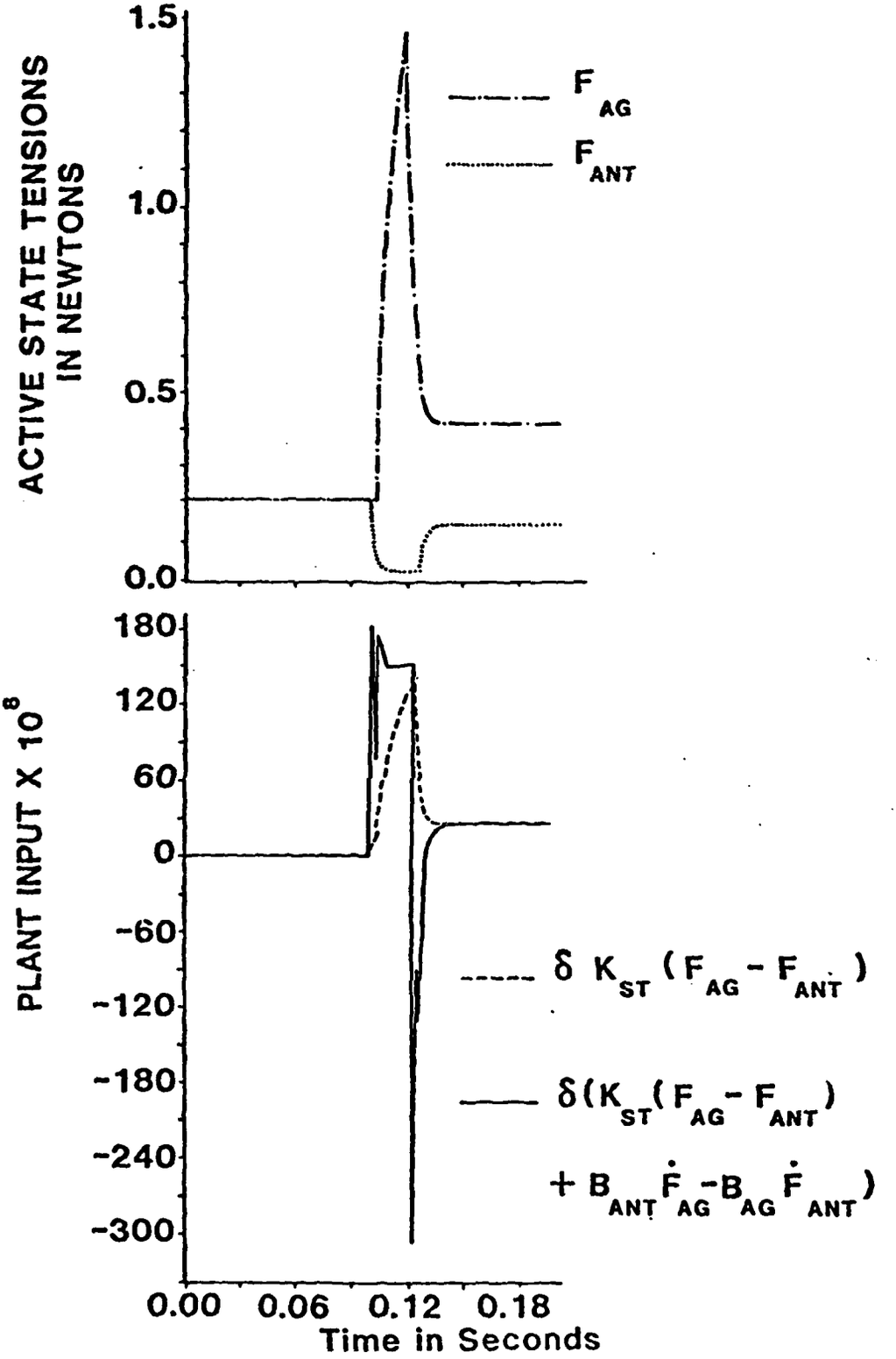
$\frac{1}{s^4+P_3s^3+P_2s^2+P_1s+P_0}$
Eye Position
ANTAGONIST FORCE GENERATOR
$SB_{AG} + K_{ST}$
H


ACTIVE STATE TENSIONS IN NEWTONS
1.5
1.0
0.5
0.0
F AG
F ANT
PLANT INPUT X 10^8
180
120
60
0
-60
-120
-180
-240
-300
δ K ST (F AG - F ANT)
δ(K ST (F AG - F ANT)
+ B ANT Ḟ AG - B AG Ḟ ANT)
0.00
0.06
0.12
0.18
Time in Seconds

APPENDIX A2

Two-point central difference analysis results on three subjects for natural and modified target trajectory saccade data.

List of Symbols

Saccade - distance the target moved

Left and Right Displacement - the distance the left and right eye moved in response to the target displacement (degrees)

I.C. - initial position of the eyeball (degrees)

TD - time from the start of the saccade to the time at peak velocity (msec)

DUR - duration of the saccade (msec)

TP - time at which the saccade started (msec)

PV - peak velocity of the saccadic eye movement (degrees/second)

DIST - eye position at the time of peak velocity (degrees)

Modified Target Trajectory Data for the Left Eye for Subject 1

SACCADE	DISPLACEMENT	IC	TP	TD	DUR	PV	DIST
5°L	5.29	.61	141	16	24	272	4.5
5°L	6.12	.54	212	17	26	342	5.0
5°R	-3.97	.48	164	11	31	-235	-1.6
5°L	6.12	.54	167	22	33	320	4.8
5°R	-4.42	.9	196	18	33	-272	-2.4
5°R	-4.49	.67	198	20	41	-240	-3.1
5°R	-4.2	.9	192	20	32	-251	-2.8
5°L	6.02	.96	193	19	26	283	5.4
5°L	6.38	1.15	238	14	28	294	4.3
5°R	-2.05	1.57	157	20	30	-176	-1.3
5°L	6.38	1.06	206	19	26	368	5.9
5°R	-3.01	1.22	141	15	18	-224	-1.5
5°R	-3.49	1.35	149	15	31	-262	-1.4
5°L	6.28	1.22	158	19	32	251	5.0

SACCADE	LEFT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST	RIGHT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST
5°L	5.56	.80	17	26	195	289	4.5	5.64	.97	18	32	194	239	3.7
5°L	6.19	.21	16	32	178	305	3.8	6.01	.26	21	48	177	268	4.3
10°L	10.15	.28	17	39	182	379	5.0	11.15	.79	18	41	182	345	5.2
10°L	10.39	.49	28	38	197	379	8.0	11.04	.57	18	42	198	319	4.2
10°L	10.72	.66	19	40	199	371	6.5	11.15	.64	25	42	199	356	7.8
15°L	14.26	.77	18	59	172	406	5.7	15.73	1.23	25	56	176	334	9.0
15°L	14.33	.68	28	58	186	387	9.7	15.64	.66	21	56	187	389	6.4
15°L	14.51	.59	21	53	212	418	6.9	15.95	.73	24	56	213	382	7.4
5°R	-4.36	.91	16	34	163	-231	-1.8	- 3.99	1.17	18	29	164	-246	-2.7
5°R	-4.85	.59	23	34	171	-215	-3.6	- 4.67	.70	19	36	169	-275	-3.2
10°R	-9.1	.47	26	50	210	-297	-5.8	- 8.63	.70	19	44	209	-356	-4.3
10°R	-9.66	.33	19	49	131	-293	-4.3	- 8.83	.59	23	50	127	-290	-4.8
10°R	BAD RECORDS													
15°R	-10.93	.77	24	70	182	-250	-4.8	-11.67	.81	20	67	180	-352	-4.5
15°R	-11.25	.70	20	55	182	-340	-4.1	-11.28	.79	19	59	180	-374	-4.2
15°R	-12.57	.52	35	66	183	-309	-6.8	-12.0	.59	19	58	186	-316	-4.2

Normal Saccade Data for Subject 1

Modified Target Trajectory Data for the Left Eye of Subject 2

SACCADE	DISPLACEMENT	IC	TP	TD	DUR	PV	DIST
5°L	4.4	.10	177	17	25	233	3.4
5°L	4.09	-.13	198	11	23	263	2.2
5°R	-4.35	-.16	165	12	23	-242	-2.6
5°L	4.01	-.31	193	10	21	242	2.4
5°R	-4.94	-.47	153	18	28	-237	-3.8
5°R	-5.05	.18	212	18	25	-280	-4.4
5°R	-4.32	-.16	175	18	34	-233	-2.9
5°L	3.24	-.34	194	13	28	229	1.6
5°L	4.04	-.31	230	9	21	229	1.3
5°R	-4.79	-.26	158	8	25	-203	-1.8
5°L	3.42	-.03	187	15	24	198	2.8
5°R	-4.09	-.05	224	15	28	-185	-2.7
5°R	-4.66	0.0	240	17	30	-237	-3.3
5°L	4.09	-.08	292	9	21	267	2.5

SACCADE	LEFT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST	RIGHT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST
5°L	3.61	- .99	18	29	174	207	1.8	4.72	.07	21	34	175	214	3.5
5°L	3.86	-.91	17	25	180	253	2.6	4.29	-.19	15	25	182	248	2.8
10°L	7.03	-1.14	18	37	176	341	2.9	8.98	.56	20	39	177	282	4.6
10°L	7.41	- .89	19	37	165	345	3.7	9.13	.26	20	39	165	304	4.6
10°L	8.52	-.76	28	37	215	341	7.0	9.57	.09	23	38	218	344	6.1
15°L	10.21	- .97	21	48	217	331	3.9	12.47	.13	27	47	221	338	7.5
15°L	10.3	-1.16	26	46	230	348	6.2	13.40	.54	25	54	227	313	6.0
15°L	11.27	-1.12	20	50	246	359		BAD DATA						
5°R	-5.30	-1.16	14	32	159	-204	-3.1	- 4.46	.0	17	28	159	-239	-3.2
5°R	-5.30	-1.03	16	33	173	-211	-3.5	- 4.33	.39	16	26	170	-273	-2.5
10°R	-10.38	-.95	24	38	168	-334	-6.9	- 9.61	-.43	25	39	163	-347	-6.1
10°R	-10.55	-1.16	21	44	167	-299	-5.0	- 9.57	-.19	26	40	167	-353	-7.1
10°R	-10.74	-1.05	25	35	173	-376	-8.4	- 9.33	.07	23	31	171	-390	-7.2
15°R	-14.66	-1.05	28	43	183	-411	-10.4	-11.69	.11	19	41	181	-406	-6.3
15°R	-14.7	-.65	21	44	200	-415	-6.6	-12.06	.11	20	37	201	-418	-7.0
15°R	-15.51	- .86	23	52	211	-352	-6.5	-12.58	.13	22	49	211	-390	-6.4

Normal Saccade Data for Subject 2

Modified Target Trajectory Data for the Left Eye of Subject 3

SACCADE	DISPLACEMENT	IC	TP	TD	DUR	PV	DIST
5°L	5.44	1.05	207	10	29	213	2.9
5°L	5.29	1.16	197	14	26	252	3.7
5°R	-3.11	1.10	191	9	29	-252	-0.8
5°L	6.1	1.19	450	11	25	271	4.4
5°R	-2.99	1.6	204	16	28	-233	-1.2
5°R	-2.53	1.77	206	11	25	-247	-0.8
5°R	-3.63	1.74	198	21	31	-281	-2.3
5°L	6.51	2.15	171	15	27	218	4.6
5°L	6.43	2.24	212	11	21	286	4.8
5°R	2.56	-2.30	191	9	26	-213	0.6
5°L	6.25	2.56	156	15	27	208	5.1
5°R	2.41	-2.24	190	10	24	-262	-0.1
5°R	2.70	-1.34	229	9	32	-199	1.0
5°L	7.30	2.62	204	12	25	233	5.2

SACCADE	LEFT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST	RIGHT DISPLACEMENT	I.C.	TD	DUR	TP	PV	DIST
5°L	6.14	2.45	15	38	165	179	4.1	4.65	.17	15	41	167	194	1.9
5°L	6.98	1.71	20	28	185	286	5.4	5.98	.06	20	26	191	248	4.9
10°L	10.62	.95	21	38	205	401	6.2	10.2	.43	18	34	209	391	5.6
10°L	10.9	2.0	21	43	203	329	6.8	10.08	.22	22	45	203	323	5.5
10°L	11.00	2.26	23	39	169	357	8.1	10.10	.34	21	42	169	344	5.4
15°L	12.90	2.69	22	45	174	373	7.6	12.03	.15	23	50	173	377	4.9
15°L	14.88	2.07	23	45	177	397	8.8	13.91	.30	22	52	176	420	6.1
15°L	15.38	2.12	21	52	194	385	7.6	14.53	.41	21	52	194	402	6.1
5°R	-2.43	2.62	21	40	170	-254	0.0	- 4.50	.26	24	39	169	-262	-3.2
5°R	- 3.6	1.86	19	33	182	-254	-2.1	- 5.27	.09	17	31	179	-312	-3.2
10°R	-6.95	2.38	29	44	169	-357	-5.0	- 8.20	.19	21	42	169	-341	-4.9
10°R	-8.02	1.79	20	38	193	-389	-4.2	- 9.24	0.00	23	42	187	-395	-5.4
10°R	-9.43	1.6	24	43	190	-341	-4.6	-10.09	.09	17	39	189	-337	-4.1
15°R	-11.6	1.86	23	44	310	-437	-5.7	-11.93	.13	26	42	308	-398	-8.2
15°R	-11.95	2.14	22	55	168	-409	-3.6	-10.64	.54	21	46	167	-427	-5.2
15°R	-12.17	2.12	25	54	169	-417	-4.2	-11.52	.32	22	47	170	-434	-6.1

Normal Saccade Data for Subject 3

OPTIMAL CONTROL OF SACCADIC EYE MOVEMENTS

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Proceedings of the Twenty-First Annual Rocky Mountain Bioengineering Symposium, Boulder, Colorado, April 1984

ABSTRACT

A new theory describing the optimal control of saccadic eye movements is proposed based on physiological considerations. The lateral and medial rectus muscle of each eye is modeled as a Voigt element connected to a series elastic element. The eyeball is modeled as a sphere connected to a viscosity and elastic element. Each of these elements is assumed to be ideal and linear. The active state tension for each muscle is modeled by a low-pass filtered pulse-step waveform. The magnitude of the agonist pulse is assumed to be a maximum constant for saccades of all sizes and only the duration of the agonist pulse affects the size of the saccade. The antagonist muscle is assumed to be completely inhibited during the period of maximum stimulation for the agonist muscle. Under these assumptions, the eyeball is directed to its destination in minimum time for saccades of all sizes. Horizontal saccadic eye movements were recorded from infrared reflection signals off the anterior surface of the cornea and then digitized. Parameter estimates are calculated for the model by using a conjugate gradient search program which minimizes the integral of the absolute value of the error squared between the model and the data. The predictions of the model are shown to be in good agreement with the data.

INTRODUCTION

Saccadic eye movements are among the fastest voluntary muscle movements the human body is capable of producing. Although the purpose for such eye movement is obvious, that is, to quickly redirect the eyeball on the target, the neuronal control strategy is not. For instance, does the word "quickly" in the previous sentence imply the most rapid movement possible, or simply a fast as opposed to a slow movement? One group of researchers investigating the neuronal control strategy concluded that each saccadic eye movement is optimally driven by a low-pass filtered pulse-step muscle force (active state tension) to achieve the final eye position in minimum time (1)(2). Their results, however, failed when applied as a consistent neuronal control mechanism for all saccadic eye movements independent of the magnitude of the saccade. In order to reach a destination in minimum time, the system should be fully stimulated, and then fully braked, regardless of the distance traversed. Under this control strategy, the only variable affecting saccade magnitude is the length of time that the system is fully stimulated. Since they assumed that saccade magnitude is both a function of the amplitude and duration of the saccadic innervation signal, and not of

duration alone, it is not an obvious minimum time controller.

In order to detail the neuronal control strategy, it is necessary to understand the effect of the measurable saccadic innervation signal on the oculomotor plant and the resultant eyeball response. Collins states that the magnitude of each saccade is determined by the amplitude and duration of the saccadic innervation signal (3). Specifically, he determined a logarithmic innervation amplitude to saccade magnitude relationship. Zee and his co-workers assumed that the amplitude and duration of the saccadic innervation signal is automatically controlled by a local feedback loop (4). The saccadic innervation signal is driven by the difference between the internal representation of the present eye position and the desired eye position. Note that these authors still hypothesize a pulse-step innervation signal, but the pulse size and duration are based on a nonlinear velocity function. If these authors are correct in their assertion, that is, saccade magnitude is a function of both the duration and the amplitude of the saccadic innervation signal (and not duration alone), then the neuronal control does not operate with an apparent minimum time strategy.

Recently, Robinson presented data that seems to contradict the previous saccadic innervation relationship of

amplitude and duration with saccade magnitude (5). Innervation seems to peak at the same height and maintain a constant but lower level throughout the saccade for the agonist muscle for any size retinal error (see Fig. 4 in Robinson). Apparently, only the duration of the agonist pulse is a function of the target displacement. A possible explanation for this disparity with other investigators recordings may lie in the type of motoneuron from which the recordings were made (3). If, in fact, Robinson's saccadic innervation signals are a correct representation, then a neuronal control strategy of minimum time still does not seem appropriate because of the higher initial firing rates.

While investigators have been able to record the innervation signal from several types of motoneurons that drive the eyeball during a saccade, they have not been able to directly measure the muscle force responsible for this movement. Collins and his co-workers have measured the tension, in vivo, at the muscle tendon during unrestrained human eye movement using a miniature "C" gauge force transducer (6). The muscle force, however, is distributed throughout the muscle and cannot be directly measured since it is modified by the visoelasticity of the muscle. Very little is known about the dynamic muscle forces generated in the antagonist-agonist pair during a saccade and their relationship to the saccadic innervation signal. Naturally,

under static conditions during fixation, the muscle forces are proportional to the innervation signal, the constant firing frequency produces a constant force. During a saccade, the agonist muscle force changes rapidly, rising to a new level approximately ten-fold higher than fixation in milliseconds, and then falling to the new fixation level. The proportional relationship that exists for innervation and muscle force during fixation is not valid during a saccade due to the effect of saturation and filtering on the innervation signal. Thus, the exact shape of the muscle waveform is uncertain. It has been observed that the muscle force lags behind the saccadic innervation signal by 8 msec due to diffusion and resequestering of Ca^{++} ions in the muscle and is typically modeled by low-pass filtering of the innervation signal (5). While very little is known about the activation and deactivation time constant due to lack of in vivo testing, Bahill has estimated their values to be between .2 and 13 msec based on the rise of the isometric force during electrical stimulation (7). When muscles are artificially stimulated, they do not develop additional tension when the stimulus frequency is above 200 Hz (5). The normal firing frequency for the agonist muscle averages approximately 800 Hz during the saccade (5). One explanation for why motoneurons fire at rates beyond which the muscle can respond is that both the antagonist and agonist

rate of change force significantly contributes to driving the eyeball to its destination (5)(8)(9). Since saccades of all sizes fire well above 200 Hz for the initial pulse phase of the trajectory, then are the saccades all driven by the same magnitude agonist muscle force pulse? Furthermore, if the magnitude of the agonist pulse is the same for any size saccade, then the only variable affecting the saccadic size is the duration of the agonist pulse. Under these conditions, the eyeball is driven to its destination in minimum time for saccades of all sizes.

In this paper we present a quantitative analysis of saccadic eye movements to support the hypothesis that the saccadic neuronal control mechanism operates to achieve final eye position in minimum time. The concepts underlying this hypothesis are: (1) the active state tension for each muscle is modeled by a low-pass filtered pulse-step waveform in which the magnitude of the agonist pulse is a maximum constant for saccades of all sizes, and (2) only the duration of the agonist pulse affects the size of the saccade. The antagonist muscle is assumed to be completely inhibited during the period of maximum stimulation for the agonist muscle.

METHODS

Data were collected after seating a subject before a target display of nine small red light emitting diodes (LED), each separated by five degrees. The subject's head was restrained by a bite-bar. The subject was instructed to follow the "jumping" target which moved from the center position to any of the other LED's and then returned to the center position. The order of the target positions as well as the time interval between displacements was randomized. Data were only recorded for the initial displacement from the center position.

Horizontal eye movements were recorded from each eye using an infrared reflection signal from the anterior surface of the cornia-scleral interface with instrumentation described by Engelken et al. (10). Signals for both eyes tracking were digitized using the analog/digitial converter of a DECLAB PDP 11/34 computer and stored in disk memory. These signals were sampled at a rate of 1000 samples per second for one-half second.

MODEL OF THE OCULOMOTOR PLANT

The diagram in Fig. 1 illustrates the mechanical components of the oculomotor plant for horizontal eye movements. The lateral and medial rectus muscle of each eye is modeled as a Voigt element connected to a series

elastic element K_{SE}. A Voigt element for the agonist muscle consists of an ideal active state tension generator F_{AG} in parallel with viscolsity element B_{AG} and length tension spring K_{LT}. The eyeball is modeled as sphere with moment of inertia J_p connected to viscosity element B_p and elastic element K_p. The passive elasticity for each muscle is included in spring K_p for ease in analysis. Each of the elements defined in the model of the oculomotor plant is ideal and linear. θ is the angle the eyeball is deviated from the primary position (looking straight ahead), and variable x is the length of the arc traversed. When the eye is at the primary position, both θ and x are equal to zero. Variables x_1 through x_4 describe the displacement from equalibrium for the stiffness elements in each muscle. Values x_{p1} through x_{p4} are the displacements from equalibrium for each of the variables x_1 through x_4 at the primary position. The total extension of the muscle from equilibrium at the primary position is x_{p1} plus x_{p2} or x_{p3} plus x_{p4} which equals approximately 3mm (11). It is assumed that the lateral and medial rectus muscles are identical, such that x_{p1} following fourth-order model was derived for the oculomotor system shown in Fig. 1 by Enderle, Wolfe, and Yates (9).

$$\delta\,(K_{ST}(F_{AG}-F_{ANT})+B_{ANT}\dot{F}_{AG}-B_{AG}\dot{F}_{ANT}) = P_4\ddot{\theta}+P_3\dot{\theta}+P_2\ddot{\theta}+P_1\dot{\theta}+P_0\theta,$$

where

$$\delta = 57.296\, K_{SE}/r,$$

$$K_{ST} = K_{SE} + K_{LT},$$

$$J = 57.296\, J_p/r^2,$$

$$B = 57.296\, B_p/r^2,$$

$$K = 57.296\, K_p/r^2,$$

$$P_4 = JB_{ANT}B_{AG},$$

$$P_3 = JK_{ST}(B_{AG} + B_{ANT}) + BB_{ANT}B_{AG},$$

$$P_2 = JK_{ST}^2 + BK_{ST}(B_{AG}+B_{ANT}) + B_{AG}B_{ANT}(K+2K_{SE}),$$

$$P_1 = BK_{ST}^2 + (B_{AG}+B_{ANT})(KK_{ST}+2K_{SE}K_{ST} - K_{SE}^2),$$

and

$$P_0 = KK_{ST}^2 + 2K_{SE}K_{ST}K_{LT}.$$

RESULTS

Representative saccadic eye movement data and model simulation results are shown in Fig. 2. A similar close agreement between the predicted saccadic eye response and the data is seen for input target displacements from -20 to 20 degrees. The simulation results are calculated using the IBM Continuous System Modeling Program for the oculomotor model in equation 1, and the following low-pass filtered pulse-step signals for the agonist and antagonist active state tensions.

$$F_{AG}(t) = F_{AGO}u(t) + (F_{AGP}-F_{AGO})(1-\exp\{-(t-t_p)/\tau_{ac}\})u(t-t_p) - (F_{AGP}-F_{AGS})(1-\exp\{-(t-t_p-t_d)/\tau_{de}\})u(t-t_p-t_d),$$

$$F_{ANT}(t) = F_{ANTO}\ u(t) - F_{ANTO}(1-\exp\{-(t-t_p)/\tau_{de}\})u(t-t_p) + F_{ANTS}(1-\exp\{-(t-t_p-t_d)/\tau_{de}\})u(t-t_p-t_d),$$

where

F_{AGO} = initial magnitude of the agonist active state tension

F_{AGP} = pulse magnitude of the agonist active state tension,

F_{AGS} = step magnitude of the agonist active state tension,

F_{ANTO} = initial magnitude of the antagonist active state tension,

F_{ANTS} = step magnitude of the antagonist active state tension,

t_p = latent period, the time interval between the target movement and the start of the eye movement

t_d = duration of the agonist pulse active state tension

τ_{ac} = activation time constant,

and

τ_{de} = deactivation time constant.

Quantitative analysis of the data was conducted in two stages. During the first stage, all model and force parameters are estimated with a conjugate gradient search program which minimizes the integral of the absolute value

stages. During the first stage, all model and force parameters are estimated with a conjugate gradient search program which minimizes the integral of the absolute value of the error squared between the frequency response and the data for the first saccadic eye movement (12). Physiological data is used to estimate all of the initial parameter estimates (6)(11). During the second stage, all subsequent saccadic eye movements are analyzed using the conjugate gradient search program in which only the latent period and the duration of the agonist pulse are optimally adjusted. All other model and force parameter values are set equal to those values computed during the first stage.

DISCUSSION

The objective of this study was to investigate the hypothesis that saccadic eye movements are generated under a time optimal neuronal control strategy for saccades of all sizes. Specifically, we postulate that the active state tension for each muscle is modeled by a low-pass filtered pulse-step waveform in which the magnitude of the agonist pulse is a maximum constant for saccades of all sizes and that only the duration of the agonist pulse affects the size of the saccade. The antagonist muscle is assumed to be completely inhibited during the period of maximum stimulation for the agonist muscle. Thus, the eyeball is

driven to its final position in minimum time for saccades of all sizes. The time optimal hypothesis is supported by the simulation results which closely match the measured saccadic eye movement data for target displacements from -20 to 20 degrees. A time optimal neuronal control strategy is also supported through physiological evidence. Experiments involving artificial stimulation of oculomotor muscles above 200 Hz indicate that such muscles do not develop additional tension (5). Since the agonist firing rate for saccadic eye movements is much above 200 Hz, then the same magnitude agonist pulse must drive the eyeball regardless of saccade size. Experimental results indicate a clear relationship between the duration of the agonist saccadic innervation signal (and corresponding active state tension) and the size of the saccade.

It is well known in optimal control theory that in order to transfer a system from point A to point B in minimum time, the system should be maximally accelerated for one-half the time interval, and then maximally decelerated for the remainder of the time interval (13). Due to the low- pass filtering on the active state tension signal and the unequal activation and deactivation time constants, the periods of acceleration and deceleration are unequal for saccadic eye movements. Evidence indicates that during the period of acceleration, the agonist pulse takes on a maximum

value regardless of the size of the saccade. During the period of deceleration, the oculomotor system is dynamically braked by the antagonist step waveform and the rate of change forces, $d F_{AG}/dt$ and $d F_{ANT}/dt$ (9). Therefore, the saccadic eye movement system is in fact, a time optimal system.

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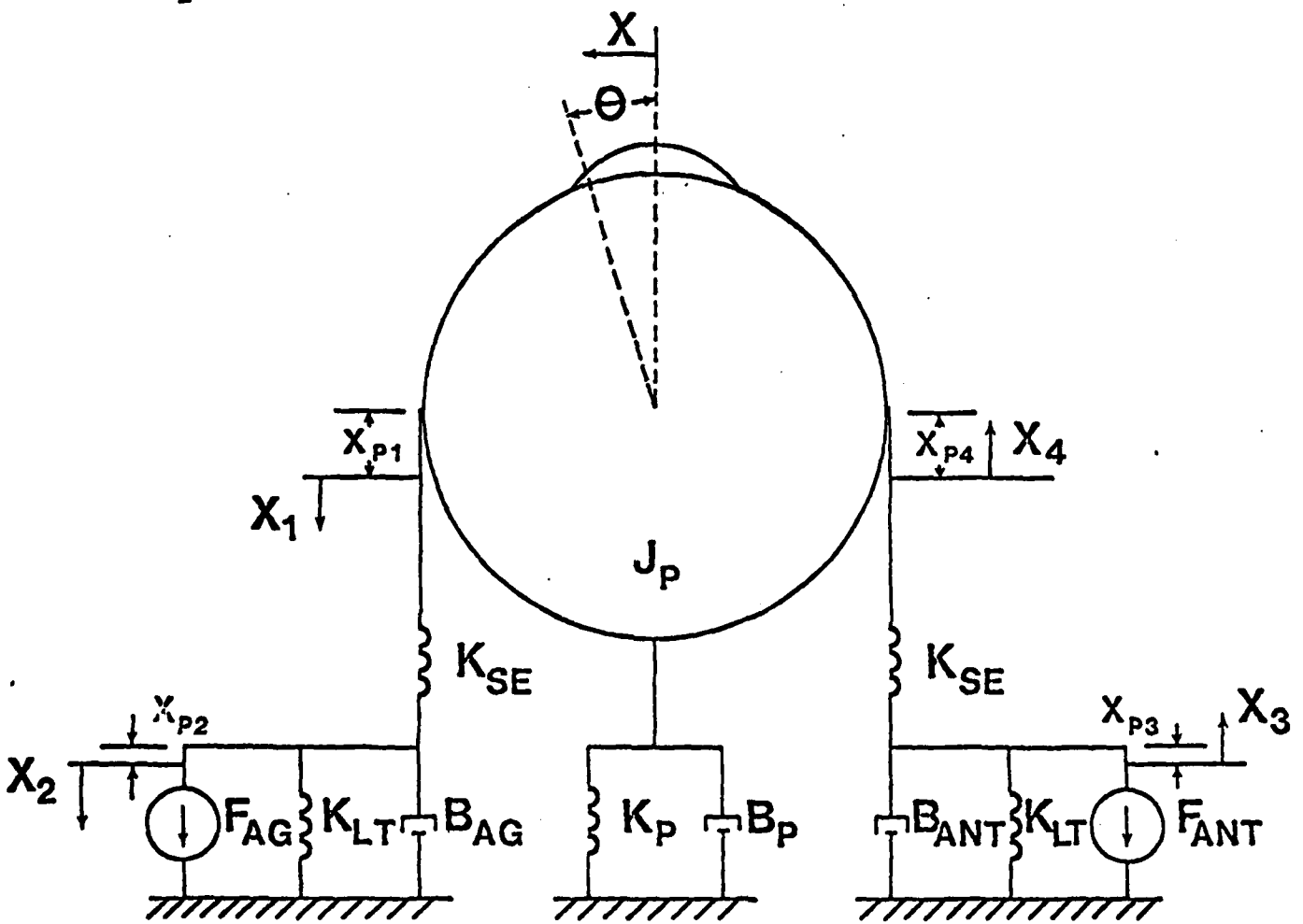

FIGURE 1. The mechanical components for the oculomotor plant at primary position.

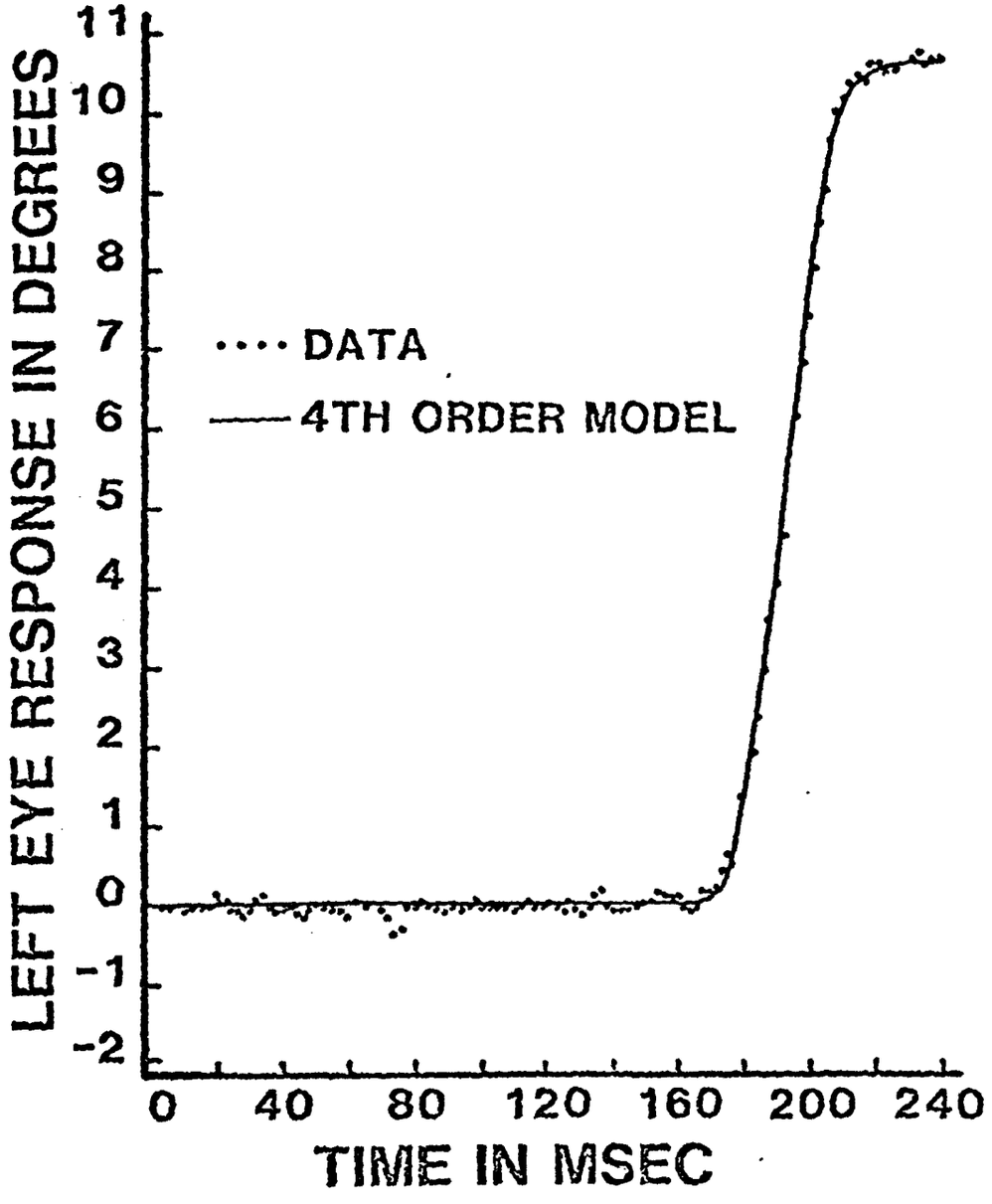

Figure 2. A 10° Saccadic Eye Movement.

A MODEL OF HORIZONTAL SACCADIC EYE MOVEMENTS

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2. Supported in part by a Grant #F49620-82-C-0035 from the Air Force Office of Scientific Research, Washington, D.C.

Proceedings of the Seventeenth Hawaii International Conference on System Sciences, Honolulu, Hawaii, January 1984.

ABSTRACT

A simple second-order model of the horizontal saccadic eye movement system is developed from physiological considerations. The lateral and medial rectus muscles of each eye are modeled as a Voigt element connected to a series elastic element. The eyeball is modeled as a sphere connected to a viscosity element and an elastic element. Each of these elements are assumed to be ideal and linear. The model is derived so that the initial conditions for the elastic elements are included in the initial agonist and antagonist muscle forces and eye position. As a result, all parameters and initial conditions can be estimated or directly measured from physiological data. The derivation is sufficiently general that no requirements are assumed for the saccadic muscle forces. Close agreement between the predicted saccadic eye response and the data was obtained for target displacements from -20 to +20 degrees with an assumed low-pass filtered pulse-step muscle force.

1. INTRODUCTION

Models of oculomotor function are important in the development of clinically useful diagnostic tools, and in understanding how the human brain controls eye movement. (1)-(8) The complexity of oculomotor models and their correlation with physiological evidence has increased since Westheimer first presented a model of saccadic eye movements in 1954. (9) Unfortunately, the complexity of these models have precluded their widespread use in most clinical studies of oculomotor function. Investigators have utilized zero-order or second order plant functions with reduced precision and little or no theoretical considerations. (1)-(3) (5) (6) (10)

This paper presents a linear second-order oculomotor plant model that is developed from physiological data. This model is a modification of the sixth-order linear homeomorphic model presented by Bahill, Latimer and Troost which retains the excellent match between the model predictions and the data. (11) The simplicity of the model and its correlation with physiological data make it ideal for use in the development of more sensitive tests of oculomotor pathology and in the description of normal oculomotor function.

2. MODEL OF OCULOMOTOR PLANT

Figure 1 illustrates the mechanical components of the oculomotor

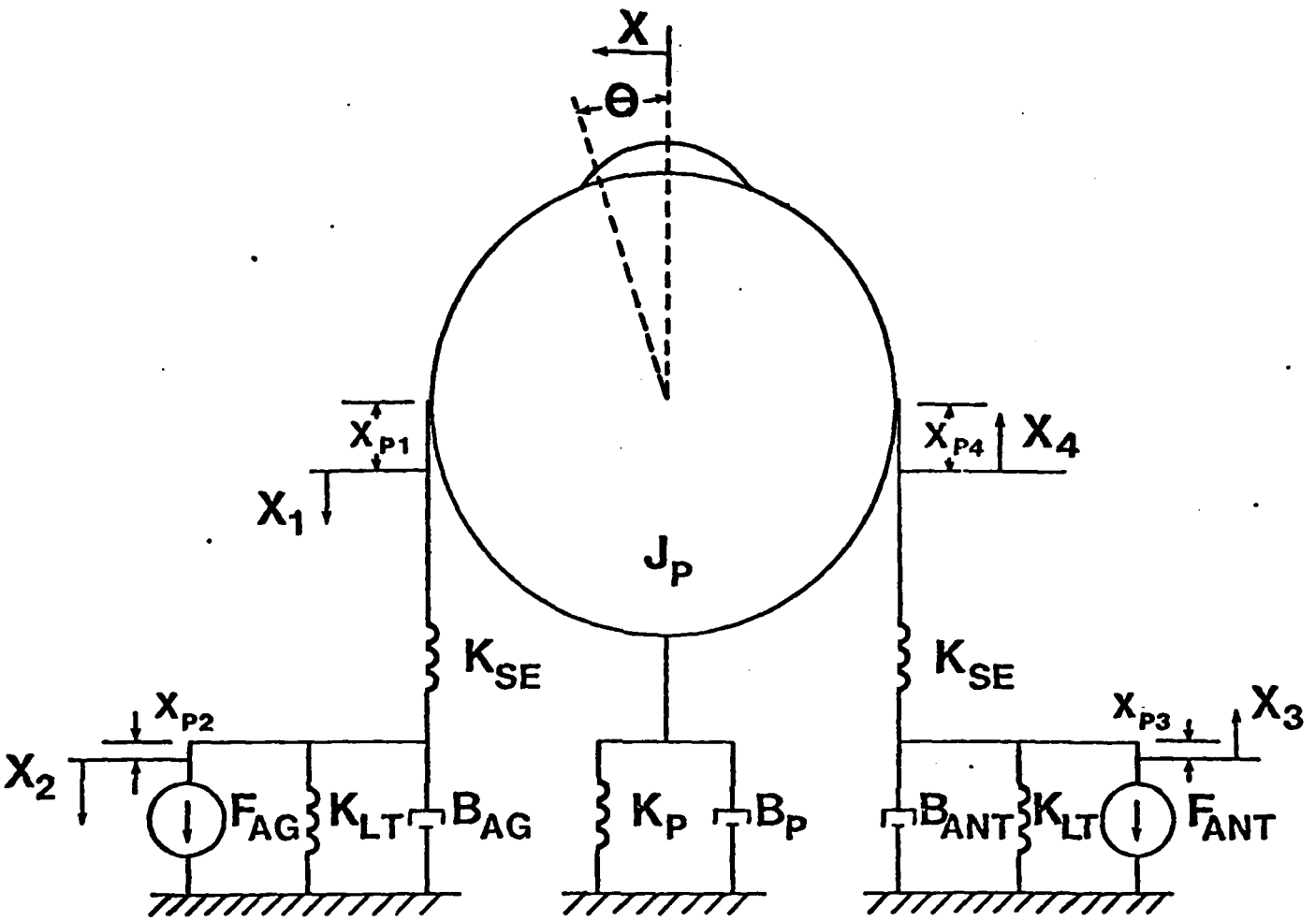

Figure 1. Mechanical components for the oculomotor plant at the primary position.

plant for horizontal saccadic eye movements. The lateral and medial rectus muscles of each eye are modeled as a Voigt element connected to a series elastic element K_{SE}. A Voigt element for the agonist muscle consists of an ideal force generator F_{AG} in parallel with viscosity element B_{AG} and length tension spring K_{LT}. The eyeball is modeled as a sphere with moment of inertia J_p connected to viscosity element B_p and elastic element K_p. The

passive elasticity for each muscle is included in spring K_p for ease in analysis.

The model presented in Figure 1 is consistent with the linear homeomorphic model except for the treatment of initial conditions. The muscles are shown to be extended from equilibrium at the primary position (looking straight ahead) consistent with physiologic evidence. (12) Variables x_1 through x_4 described the displacement from equilibrium for the stiffness elements in each muscle. Values x_{p1} through x_{p4} are the displacements from equilibrium for each of the variables x_1 through x_4 at the primary position. The total extension of the muscle from equilibrium at the primary position is x_{p1} plus x_{p2} or x_{p3} plus x_{p4} which equals approximately 3mm. (12) θ is the angle the eyeball is rotated from the primary position, and variable x is the length of the arc traversed. Each of the elements defined in Figure 1 is ideal and linear.

A set of three linear equations is written to describe the oculomotor plant by summing the forces acting at junctions 2 and 3 and the torques acting on the eyeball.

$$F_{AG} = K_{LT}x_2 + B_{AG}\dot{x}_2 + K_{SE}(x_2 - x_1), \quad (1)$$

$$F_{ANT} = K_{SE}(x_4 - x_3) - K_{LT}x_3 - B_{ANT}\dot{x}_3, \quad (2)$$

and

$$rK_{SE}(x_2 + x_3 - x_1 - x_4) = J_p\ddot{\theta} + B_p\dot{\theta} + K_p\theta, \quad (3)$$

where r equals the radius of the eyeball with a value of approximately 11 mm. Equations 1 through 3, which described the trajec-

tory of the eyeball during horizontal saccadic movement, are reduced to the following equation

$$\delta(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}\dot{F}_{AG} - B_{AG}\dot{F}_{ANT}) =$$
$$\dddot{\theta} + P_3\ddot{\theta} + P_2\ddot{\theta} + P_1\dot{\theta} + P_0\theta. \quad (4)$$

Details of the derivation and the parameter description are contained in the Appendix. This fourth-order model is derived with no constraints placed on the type of force used to drive the eyeball to its final position, such as a pulse-step force. (13) An assumption has been made that there is a constant force which acts on the eyeball until it moves. This notion is supported by studies of the discharge patterns of motoneurons. (14)

It is possible to reduce the order of the system model in equation 4 by approximating it with a second-order model. (15)-(17) This method involves minimizing the frequency response differences between the second-order system model and the original system model. It is desired to maintain the numerator of the transfer function of the original model in the second-order model for ease in analysis. This is accomplished by allowing only the denominator of the second-order model to be subject to approximation. The second-order model describing horizontal saccadic movement is

$$\delta(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}\dot{F}_{AG} - B_{AG}\dot{F}_{ANT}) =$$
$$d_2\ddot{\theta} + d_1\dot{\theta} + P_0\theta, \quad (5)$$

where

$$d_2 = (2P_0 - 2P_1P_3 + P_2^2)^{1/2}, \quad (6)$$

and

$$d_1 = (-2P_2P_0 + P_1^2 + 2P_0d_2)^{1/2}. \quad (7)$$

The eigenvalues for the second-order model could have been estimated using the two dominant poles of the fourth-order model. However, this would have resulted in less precision since the third pole affects the frequency response commencing at approximately 30 radians per sec.

3. RESULTS

Data were collected after seating subject before a target display of nine small red light-emitting diodes (LED), each separated by five degrees. The subject's head was restrained by a bite-bar. The subject was instructed to follow the "jumping" target which moved from the center position to any of the other LED's and then returned to the center position. The order of the target positions as well as the time interval between displacement was randomized. Data were only recorded for the initial displacement from the center position.

Horizontal eye movements were recorded from each eye using an infrared reflection signal from the anterior surface of the cornea-scleral interface with instrumentation described by Engelken et al. (18) Signals for both eyes tracking were digitized using the analog/digital converter of a DECLAB PDP 11/34 computer and stored in disk memory. These signals were sampled at a rate of 1000 samples per second for one-half second.

All parameters in the model are estimated with a conjugate gradient search program which minimizes the integral of the absolute value of the error squared between the frequency response of the model and the data. (19) Additionally, the latent period, the time interval between the target movement and the start of the eye movement, is also estimated. A low-pass filtered pulse step signal is assumed for the agonist and antagonist muscle forces. (20) Both muscles has the same activation and deactivation time constants. Physiological data is used to estimate all of the initial parameter estimates. (12) (21)

The estimated eigenvalues of the fourth-order oculomotor plant correlated well with their physiologically derived values. Table I shows the estimated parameter values describing both the fourth-order and the second-order oculomotor plant model.

TABLE I

Parameter values for the second-order and the fourth-order oculomotor plant models

$$P_0 = 7.56 \times 10^8$$
$$P_1 = 5.152 \times 10^7$$
$$P_2 = 5.948 \times 10^5$$
$$P_3 = 1.726 \times 10^3$$
$$d_1 = 4.890 \times 10^7$$
$$d_2 = 4.213 \times 10^5$$

Equations 6 and 7 are used to compute the parameter values for the second-order model. Close agreement between the predicted

saccadic eye movement response for both the fourth-order and the second-order models and the data is seen for input target displacements from -20 to 20 degrees. Figure 2 illustrates the observed saccadic eye movement response for a 10 degree movement and the predicted eye movement responses.

4. DISCUSSION

The objective of this study was the development of a physiologically based second-order model of the saccadic eye movement system that is potentially of value in evaluating normal and pathological oculomotor function. We feel we have accomplished this objective as illustrated in the previous sections. The model is straightforward without obscuring details and utilizes a minimum of assumptions concerning the muscle forces. The second-order model performed as well as the more complex fourth-order model and closely matches the eye movement data.

The derivation of the model is sufficiently general that no assumptions are made for the saccadic muscle forces, and the model is simple enough to be easily incorporated in oculomotor studies with little loss of precision. For example, investigators may utilize this model when examining alternatives to the pulse-step muscle force, a muscle force that is a crude approximation to the actual waveform. (14) The mechanical components of the model are connected such that the rate of change of the muscle forces together with the actual muscle forces drive the

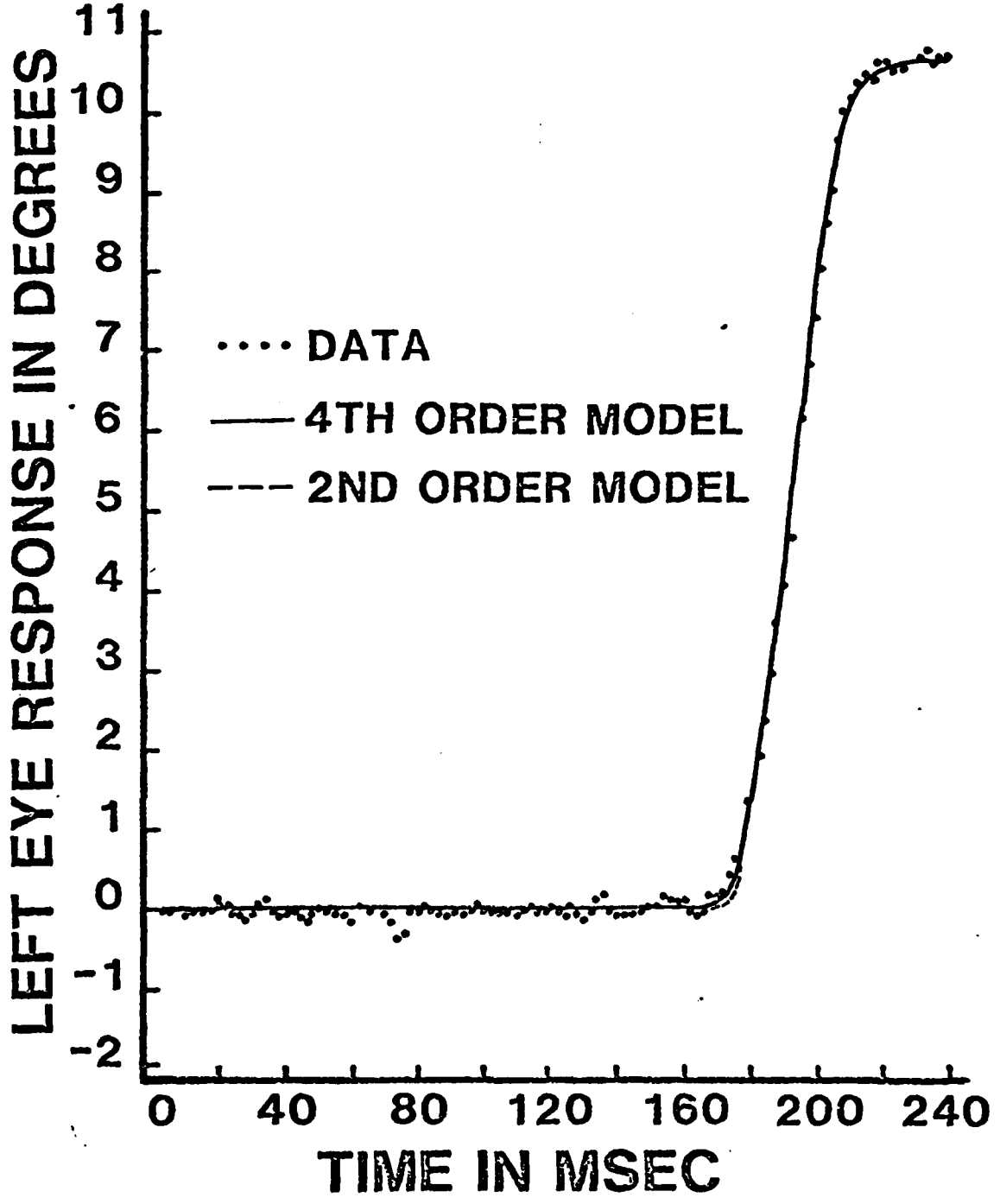

Figure 2. Time response for a 10 degree saccadic eye movement. The prediction of the second-order model was shifted forward in time for best fit for the model.

globe to its final position. This would, in part, explain why the motoneurons fire at rates beyond which the muscle can respond. (14)(22) While there is a saturation phenomenon, the

extremely high initial burst of motoneuronal firing causes the rate of change muscle forces to move the eyeball much more quickly. Additionally, the rate of change of the muscle forces act as a dynamic brake which is slowly reduced to zero as the eyeball reaches its destination.

Both the second-order model and the fourth-order model are derived such that the initial conditions for the series elastic and length tension elements are included in the estimates of the initial agonist and antagonist muscle forces and eye position. As a result, all parameters and initial conditions in these models are either directly measured or estimated from physiological data.

While the second-order model is a approximation of the fourth-order model no loss of precision is evident in the simulation studies as illustrated in Figure 1, and both predictions match the data extremely well. The approximation involved minimizing the frequency response difference between the second-order and the fourth-order models. Naturally, the simulation results for the second-order and fourth-order model will diverge for the higher order derivative terms such as eyeball acceleration and jerk. The divergence is especially prominent during the muscle force transition period, that is, during the initial rise and fall of the pulse signal. This is due to the "low pass filtering" effect of the extra two poles in the fourth-order model. One wonders whether such differences are a characterization of

muscle forces that are not truly representative of their waveforms, or an inadequate description of the oculomotor plant function.

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APPENDIX

OCULOMOTOR PLANT DERIVATION

A linear oculomotor plant model consistent with physiologic evi-

dence is to be developed in this section. It is assumed that the lateral and medial rectus muscles are identical, such that x_{P1} equals x_{P4} and x_{P3} equals x_{P2}. Furthermore when binocular tracking of targets is involved, one eye, and perhaps both eyes may not be at the primary position at the onset of movement. That case will also be considered here. The following relationships evident from Figure 1 exist among variables x_1, x_4, θ and x.

$$x_1 = x - x_{P1}, \tag{A1}$$

$$x_2 = x + x_{P4}, \tag{A2}$$

and

$$\theta = 57.296\ x/r. \tag{A3}$$

The initial condition is assigned to variables x_1, x_4, and θ through x; that is, variable x at time 0, x(0), and equations A1 through A3. Initial conditions are also assigned to variables x2 and x_3 as $x_2(0)$ and $x_3(0)$. At primary position x(0) equals zero, x_2 equals $-x_{P2}$ and x_3 equals x_{P3}.

Equations 1 through 3 are written as

$$F_{AG} = K_{ST}x_2 + B_{AG}\dot{x}_2 + K_{SE}(x_{P1} - x), \tag{A4}$$

$$F_{ANT} = K_{SE}(x + x_{P1}) - K_{ST}x_3 - B_{ANT}\dot{x}_3, \tag{A5}$$

and

$$K_{SE}(x_2 + x_3 - 2x) = J\ddot{x} + B\dot{x} + Kx, \tag{A6}$$

where

$$K_{ST} = K_{SE} + K_{LT},$$

$$J = 57.296\ J_p/r^2$$

$$B = 57.296\ B_p/r^2,$$

and

$$K = 57.296\ K_p/r^2.$$

Equations A4 through A6 are written as one equation by taking the Laplace transform of each equation and eliminating variables x_2 and x_3 yielding

$$K_{SE}(SB_{ANT} + K_{ST})L\{F_{AG}\} - K_{SE}(SB_{AG} + K_{ST})L\{F_{ANT}\} + IC = (C_4S^4 + C_3S^3 + C_2S^2 + C_1S + C_0)X(S), \quad (A7)$$

where

$$C_4 = JB_{ANT}B_{AG},$$
$$C_3 = JK_{ST}(B_{AG} + B_{ANT}) + BB_{ANT}B_{AG},$$
$$C_2 = JK_{ST}^2 + BK_{ST}(B_{AG} + B_{ANT}) + B_{AG}B_{ANT}(K+2K_{SE}),$$
$$C_1 = BK_{ST}^2 + (B_{AG} + B_{ANT})(KK_{ST} + 2K_{SE}K_{ST} - K_{SE}^2),$$
$$C_0 = KK_{ST}^2 + 2K_{SE}K_{ST}K_{LT},$$

and

$$IC = K_{SE}^2(B_{AG} - B_{ANT})x_{P1} - K_{SE}B_{AG}(SB_{ANT} + K_{ST})x_2(0) + K_{SE}B_{ANT}(SB_{AG} + K_{ST})x_{P3} + (P_4S^3 + P_3S^2 + P_2S + BK_{ST}^2)x(0). \quad (A8)$$

Simplification of the expression for IC is obtained from the steady state conditions of equations A4 through A6, that is

$$F_{AG}(0) = K_{ST}x_2(0) + K_{SE}(x_{P1} - x(0)), \quad (A9)$$

$$F_{ANT}(0) = K_{SE}(x(0) + x_{P1}) - K_{ST}x_3(0), \quad (A10)$$

and

$$x_2(0) + x_3(0) = (K/K_{SE} + 2)\, x(0). \quad (A11)$$

These three equations are used to completely remove the initial conditions $x_2(0)$ and $x_3(0)$ from equation A8, resulting in

$$IC = K_{SE}(B_{AG}F_{AG}(0) - B_{ANT}F_{ANT}(0)) + (C_4S^3 + C_3S^3 + C_2S + C_1 - C_0(B_{AG} + B_{ANT})/K_{ST})x(0). \quad (A12)$$

The equation describing the oculomotor plant is now written as

$$K_{SE}(SB_{ANT} + K_{ST})L\{F_{AG}\} - K_{SE}(SB_{AG} + K_{ST})L\{F_{ANT}\} + K_{SE}(B_{AG}F_{AG}(0) - B_{ANT}F_{ANT}(0)) - C_0(B_{AG} + B_{ANT})x(0)/K_{ST} = (C_4S^4 + C_3S^3 + C_2S^2 + C_1S + C_0)X(S) - (C_4S^3 + C_3S^2 + C_2S + C_1)\, x(0). \quad \text{(A13)}$$

Further simplification of equation A13 is obtained by assuming that the eye is undergoing a horizontal saccadic movement. Regardless of the type of force used to drive the eyeball to its final position, for example, a pulse-step force, there is a constant force acting on the eyeball until it moves. To be more explicit, the agonist and antagonist forces are decomposed into the following functions

$$F_{AG} = \begin{cases} F_{AG}(0) & t < t_d \\ F_{AG}' + F_{AG}(0) & t \geq t_d \end{cases}, \quad \text{(A14)}$$

$$F_{ANT} = \begin{cases} F_{ANT}(0) & t < t_d \\ F_{ANT}' + F_{ANT}(0) & t \geq t_d \end{cases}, \quad \text{(A15)}$$

where t_d is the time at which the eye starts to move. F_{AG}' and F_{ANT}' are zero for t less than t_d. Substituting expressions A14 and A15 into equation A13 yields

$$K_{SE}K_{ST}(F_{AG}(0) - F_{ANT}(0))/S + K_{SE}(SB_{ANT} + K_{ST})L\{F_{AG}'\} - K_{SE}(SB_{AG} + K_{ST})L\{F_{ANT}'\} = (C_4S^4 + C_3S^3 + C_2S^2 + C_1S + C_0)X(S) - (C_4S^3 + C_3S^2 + C_2S + C_1)\, x(0). \quad \text{(A16)}$$

This result follows since

$$SK_{SE}B_{ANT}F_{AG}(0)/S - SK_{SE}B_{AG}F_{ANT}(0)/S + K_{SE}(B_{AG}F_{AG}(0) - B_{ANT}F_{ANT}(0)) = C_0(B_{AG} + B_{ANT})\, x(0)/K_{ST}.$$

Note that F_{AG}' and F_{ANT}' are identical to the original agonist

and antagonist forces, respectively, except that the initial forces, $F_{AG}(0)$ and $F_{ANT}(0)$, are removed. Equation A16 is rewritten by utilizing expressions A14 and A15

$$K_{SE}K_{ST}L\{F_{AG} - F_{ANT}\} + SK_{SE}L\{B_{ANT}F_{AG}' - B_{AG}F_{ANT}'\} =$$
$$C_4S^4 + C_3S^3 + C_2S^2 + C_1S + C_0)\ X(S) -$$
$$(C_4S^3 + C_3S^2 + C_2S + C_1)\ x(0). \qquad \text{(A17)}$$

By transforming equation A17 back into the time domain and noting that

$$\frac{dF_{AG}}{dt} = \frac{dF_{AG}'}{dt} \ ,$$

and

$$\frac{dF_{ANT}}{dt} = \frac{dF_{ANT}'}{dt} \ ,$$

we have

$$K_{SE}(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}F_{AG} - B_{AG}F_{ANT}) =$$
$$C_4x + C_3x + C_2x + C_1x + C_0x.$$

Since data are available for the angle of the eyeball with respect to the primary position, the angle θ is used instead of x which yields

$$S(K_{ST}(F_{AG} - F_{ANT}) + B_{ANT}F_{AG} - B_{AG}F_{ANT}) =$$
$$\theta + P_3\theta + P_2\theta + P_1\theta + P_0\theta. \qquad \text{(A18)}$$

where

$$S = 57.296\ K_{SE}/rC_4,$$
$$P_3 = C_3/C_4,$$
$$P_2 = C_2/C_4,$$
$$P_1 = C_1/C_4,$$

and $P_0 = C_0/C_4$.

ESTIMATION OF SACCADIC EYE MOVEMENT MUSCLE FORCES USING SYSTEM IDENTIFICATION TECHNIQUES

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System identification techniques were used to estimate muscle forces during saccadic eye movements. Horizontal eye movements were digitally recorded from infrared reflection signals off the anterior surface of the cornea. The measured transfer function of the oculomotor system was calculated from the fast-eye frequency response to a step target displacement. A fourth-order linear model described the mechanical components of the eyeball and muscle tissue and a low-pass filtered pulse-step signal described the developed muscle force. Physiological data was used to estimate the initial parameter estimates, including initial conditions for the series-elastic and length-tension elastic elements. Final parameter estimates were calculated for the model by using a conjugate gradient search program which minimizes the integral of the absolute value of the error squared between the model and the data. The estimated agonist pulse has a duration approximately equal to the length of the saccade and a target displacement dependent magnitude of less than 100 grams for all eye movements. Time constants for activation and deactivation were estimated to be 3 and 4 msec respectively. Steady state muscle force estimates appeared to be in good agreement with the length-tension curves.

Research sponsored by the Air Force Office of Scientific Research/AFSC, United States Air Force, under Contract F49620-82-C-0035. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

Proceedings of the Second Southern Biomedical Engineering Conference,
San Antonio, Texas, September 1983.

ESTIMATION OF SACCADIC EYE MOVEMENT MUSCLE FORCES USING SYSTEM IDENTIFICATION TECHNIQUES

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INTRODUCTION

The present investigation utilizes system identification techniques to estimate muscle forces during horizontal saccadic eye movements. Recently, Bahill, Latimer, and Troost (1980) presented a fourth order linear oculomotor model that accurately described saccadic eye movement. Parameters were estimated using a time domain parameter estimation routine (Latimer and Bahill, 1979). The large and brief estimated agonist muscle force, however, was not consistent with physiological evidence (Robinson, 1981). This project uses frequency response methods to estimate muscle forces on a fourth order oculomotor plant during saccadic eye movements.

METHODS

The subject was seated before a target display of nine small red light-emitting diodes (LED) each separated by five degrees. The subject was instructed to follow the jumping target which moved from center position to any one of the LED's and then returned to center position. The order of the target positions as well as the time interval between displacements was random. Data was recorded only for the initial displacement from center position. Horizontal eye movements were recorded from the infrared reflection signal off the anterior surface of the cornea from instrumentation similar to that described by Young and Sheena (1975). Signals for both eyes tracking were recorded using the analog/digital converter of a DECLAB PDP 11/34 computer and stored in disk memory. These signals were sampled at a rate of 1000 samples per second for one-half second.

Model of the Oculomotor Plant

The diagram in Fig. 1 illustrates the mechanical components of the oculomotor plant for horizontal eye movements. The lateral and medial rectus muscles were modeled as a voigt element in series with spring K_{SE}. The passive elasticity from each muscle was included in spring Kp. This model is consistent with the linear homeomorphic model presented by Bahill, Latimer, and Troost (1980) except for the treatment of the initial conditions. The muscles are shown to be extended from equilibrium position at primary position consistent with physiological evidence (Robinson and co-workers, 1969). The following fourth order model was derived for the oculomotor system shown in Fig. 1.

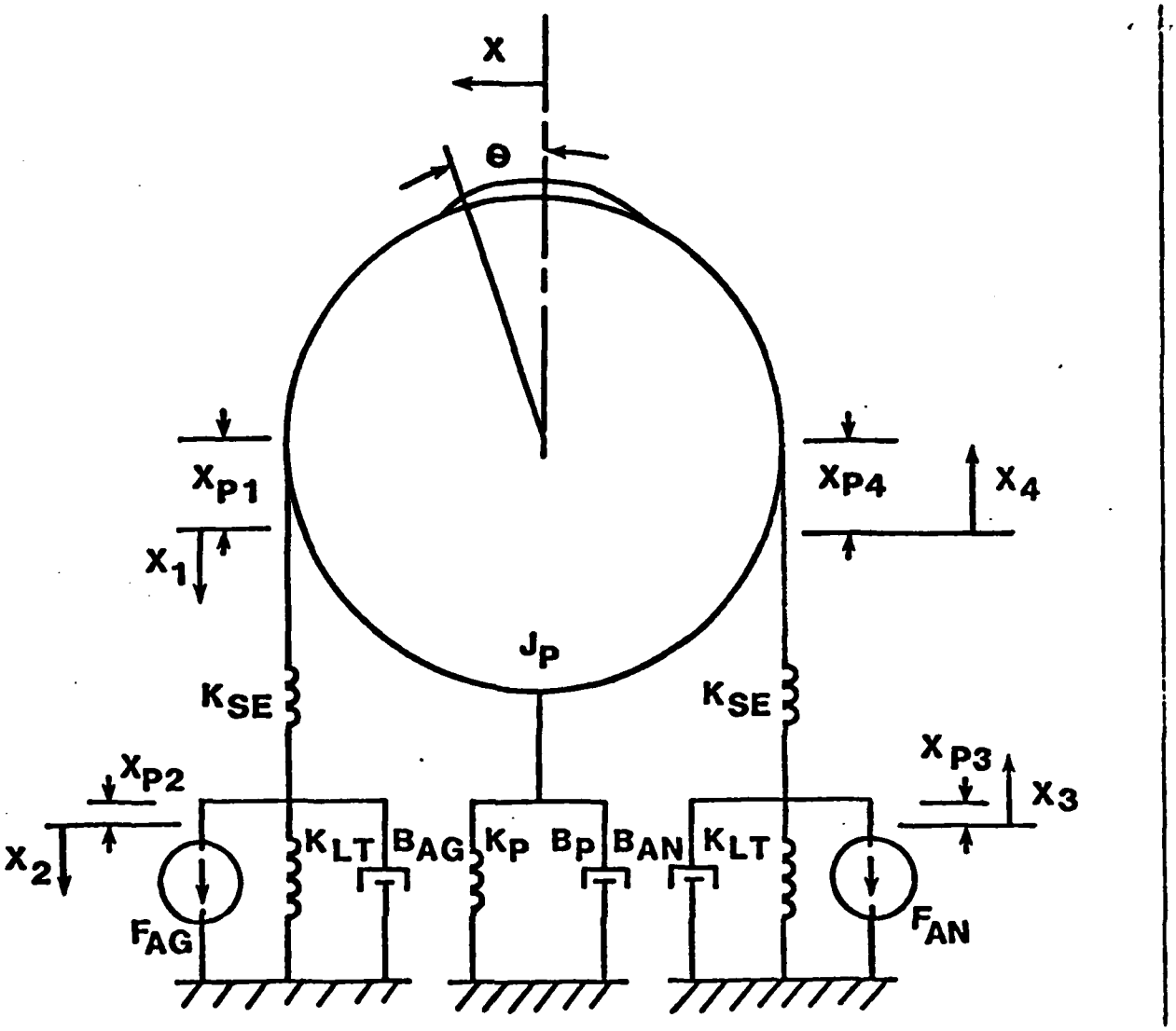

Fig. 1. Mechanical components for oculomotor plant.

$$K_{SE}\,(B_{AN}\dot{F}_{AG} - B_{AG}\dot{F}_{AN} + (K_{SE} + K_{LT})\,(F_{AG} - F_{AN})) = P_4\ddot{\ddot{X}} + P_3\dddot{X} + P_2\ddot{X} + P_1\dot{X} + P_0X, \quad (1)$$

where $P_4 = D\,J_P B_{AN} B_{AG}$,

$P_3 = D\,J_P(B_{AN} + B_{AG}) + D\,B_P B_{AN} B_{AG}$,

$P_2 = D\,J_P K_{ST}^2 + D\,B_P K_{ST}(B_{AN} + B_{AG}) + B_{AN}B_{AG}(D\,K_P + 2K_{SE})$,

$P_1 = D\,B_P K_{ST}^2 + (D\,K_P K_{ST} + 2K_{SE}K_{ST} - K_{SE}^2)\,(B_{AN} + B_{AG})$,

$P_0 = D\,K_P K_{ST}^2 + 2K_{SE}K_{ST}K_{LT}$,

$K_{ST} = K_{SE} + K_{LT}$,

r = radius of the eyeball,

and $D = 57.296/r^2$.

Equation 1 was derived such that the initial conditions for the series-elastic and length-tension elastic elements were included in the initial agonist and antagonist forces and eye position. This result means that all parameters and initial conditions can be estimated from physiological data.

System Identification Technique

The system identification technique is a frequency response method. Ideally, the transfer function is equal to the ratio of the Fourier transform of the output to the system input. For the oculomotor system, the transfer function was calculated from the fast eye response to a step target displacement. Because of unequal time delays and eye displacements from saccade to saccade for the same target displacement, it was not possible to use averaging techniques to reduce the effects of measurement noise. Fortunately, the measurement noise was small relative to the input and output signals, so the second method described by Otnes and Enochson (1972) was used. The fast eye response measurements were first filtered using a Butterworth low-pass filter with a half-power point at 125 hertz. Transforming the filtered measurements directly by the Fast Fourier algorithm resulted in distortion due to truncation, since the signal did not go to zero at steady state. This was circumvented by subtracting the steady state value from each sample, passing this signal through a Kaiser window, packing with zero's, and Fast Fourier transforming the modified signal. The Fourier transform of the fast eye response was equal to the Fast Fourier transform of the modified signal plus the Fourier transform of the unit step with amplitude equal to the steady state value. The input signal was the Fourier transform of the unit step function with amplitude equal to the steady state value. Parameter estimates for the oculomotor model were calculated using the conjugate gradient search program similar to Seidel's (1975), which minimizes the integral of the absolute value of the error squared between the model and the data. A low-pass filtered pulse-step signal described the developed muscle force. Physiological data was used to estimate the initial parameter estimates (Collins, O'Meara, and Scott, 1975; Latimer and Bahill, 1979; Robinson and co-workers, 1969).

RESULTS

A close agreement between the predicted saccadic eye response and the data was seen for input target displacements from -20 to 20 degrees. All parameters in equation were estimated with the parameter estimation routine. The eigenvalues of the oculomotor plant correlated well with their physiological data derived values. The estimated agonist pulse had an effective duration equal to the length of the saccade. The actual target displacement dependent pulse width was less than the saccade duration, but because of the low pass filtering, effectively equalled the saccade duration. The agonist pulse magnitude was also dependent on target displacement, with estimated values of less than 100 grams for all eye movements. The antagonist muscle force was assumed to have the same pulse width as the agonist muscle force and a magnitude of zero. Steady state or step muscle force estimates appeared to be in good agreement with the length-tension curves reported by Robinson (1981). Time constants for activation and deactivation were estimated to be approximately 10 and 5 msec, respectively. Shown in Fig. 2 and 3 are the time and frequency response for a 10° movement.

DISCUSSION

System identification techniques were used to estimate the eigenvalues of the oculomotor plant and muscle forces during saccadic eye movements. While the eigenvalues, pulse width, pulse height, and steady state muscle forces appear to be consistent with the physiological data and estimates of other investigators (Bahill, Latimer, and Troost, 1980; Collins, O'Meara, and Scott, 1975; Robinson and

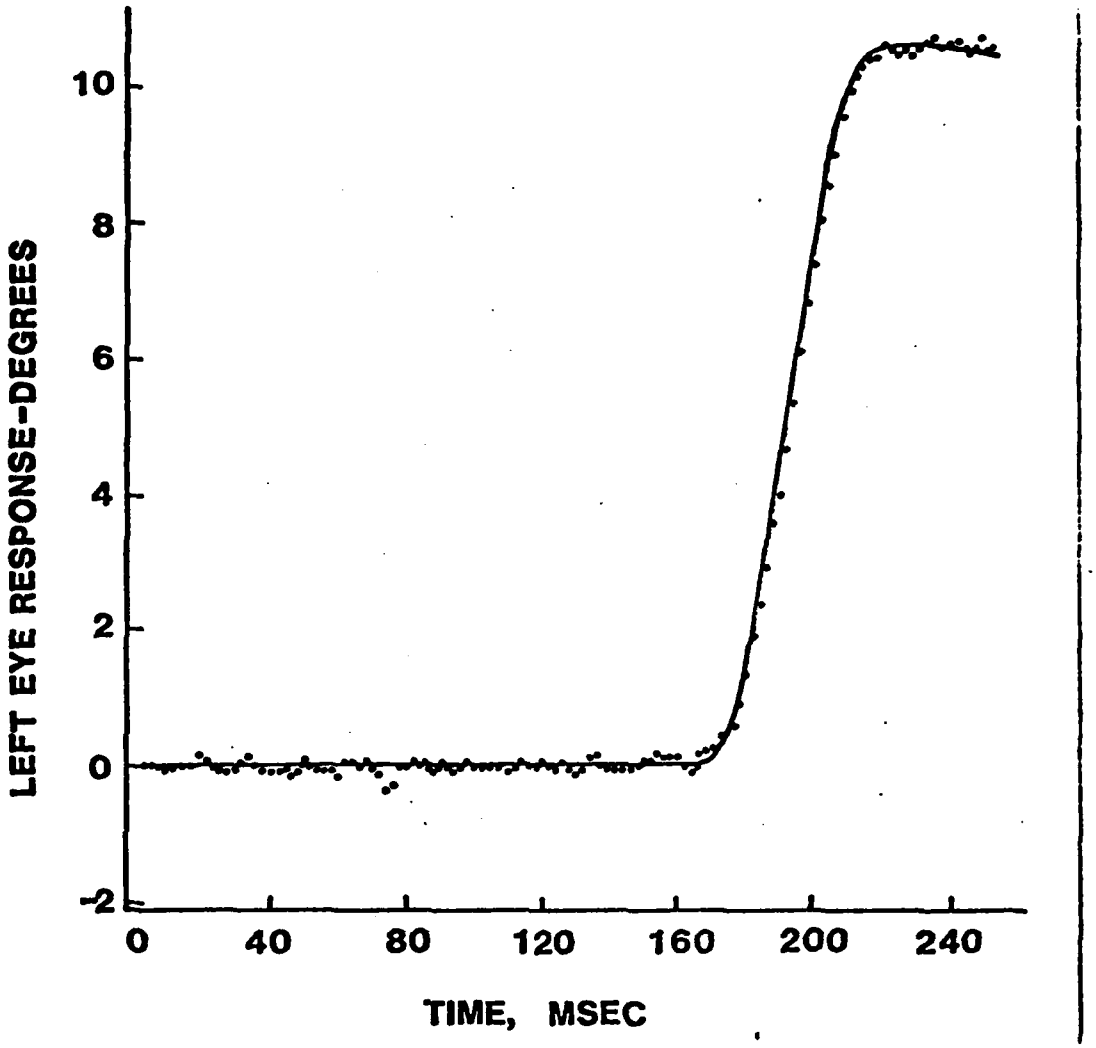

Fig. 2. Time response for a 10° eye movement.

co-workers, 1969), the estimates of the time constants appear inconsistent with the observations reported by Robinson (1981). One explanation is that the developed muscle force is not a low-pass filtered pulse step but a more complex signal. While the time response indicates an excellent fit between model and data, there appears a more significant lack of fit in the magnitude frequency response. This lack of fit may be indicative of a more complex muscle force signal.

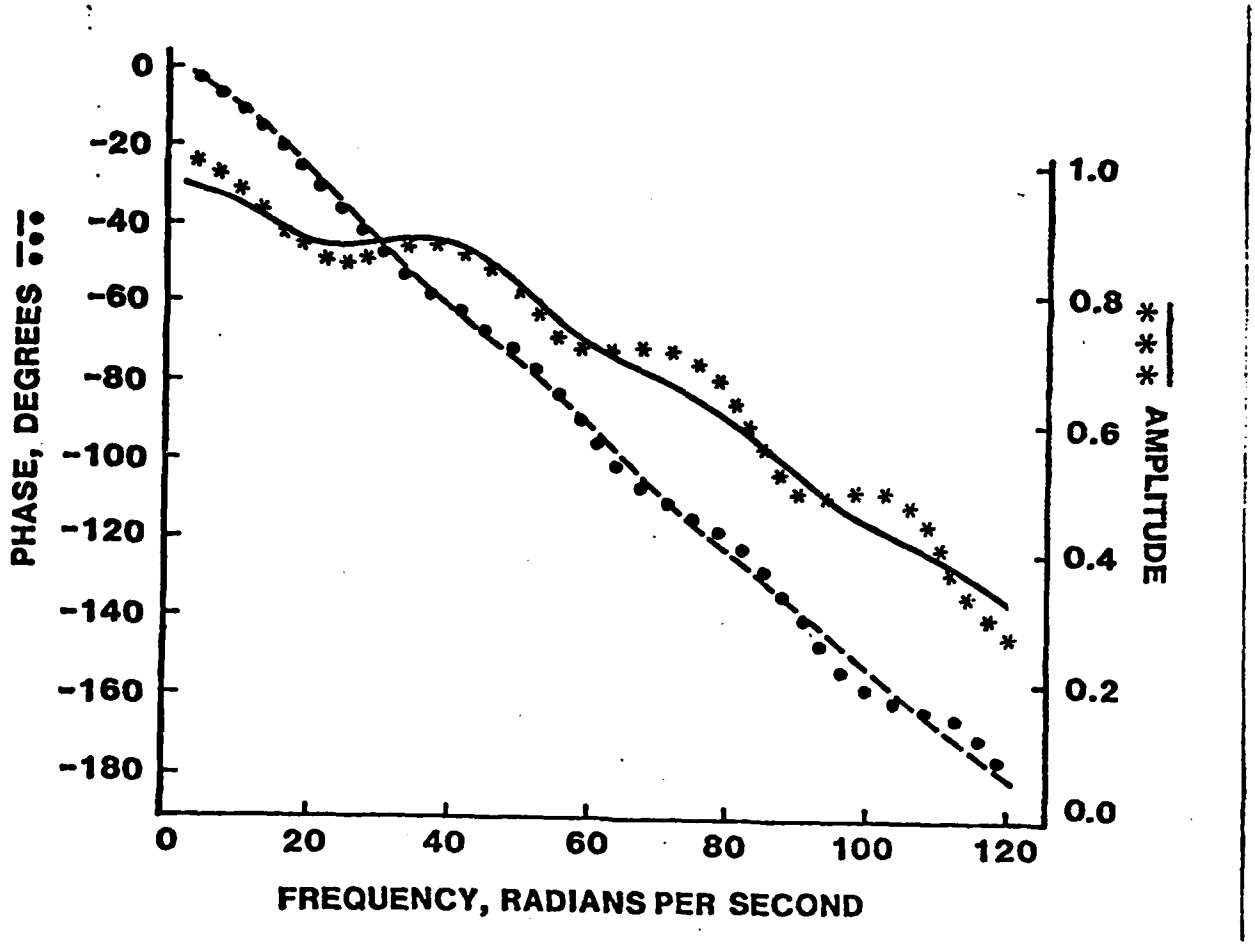

Fig. 3. Frequency response for a 10° eye movement.

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